RC NOTCH FILTERS OF THE GOLDMAN TYPE

by

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TABLE OF CONTENTS

TERMINOLOGY

A notch filter's output frequency spectrum consists of a single minimum in its frequency response. Usually it is required that the notch amplitude be zero; this leads to a zero notch filter. Two further characterizations are:

(a) An equal amplitudes notch filter has equal amplitudes at zero frequency and infinite frequency.

(b) An unequal amplitudes notch filter has unequal amplitudes at zero and infinite frequencies in its frequency response curve.

INTRODUCTION

Notch networks are important in connection with feedback amplifier problems. In the past, several notch networks with equal amplitudes and unequal amplitudes at zero and infinite frequencies have been considered. The purpose of this paper is to generalize networks of the Goldman (4) type and determine conditions for existence of the notch frequencies.

A GENERAL NOTCH FILTER OF THE GOLDMAN TYPE

A general notch filter resistance-capacitance network shown in Fig. 1 is bisected into half-sections, which are shown in Fig. 2 and Fig. 3.

If A is the short-circuit input impedance of the half-section and if B is the open-circuit input impedance of the half-section, then subsequent calculations yield

$$A = \frac{RCrs^2 + (RC + RCr + r)s + (r + 1)}{(RCr + rC)s^2 + (RC + rC + C + r)s + 1} \qquad (1)$$

$$B = 1 + \frac{1}{s} = \frac{s + 1}{s} \qquad (2)$$

Bartlett's representation theorem for symmetric networks yields the voltage transfer function T(s). Actual calculation yields

$$T(s) = \frac{B - A}{B + A}$$

$$= \frac{\left[rCs^3 + (2rC + C)s^2 + (RC + rC + C)s + 1\right]}{\left[(2RCr + rC)s^3 + (2RCr + 2RC + 2rC + 2r + C)s^2 + (RC + rC + C + 2r + 2)s + 1\right]} \qquad (3)$$

EQUAL AMPLITUDES NOTCH FILTER OF THE GOLDMAN TYPE

Referring to Eq. (3) of the general notch filter with $R = 0$, one obtains the transfer function $T_1(s)$.

$$T_1(s) = \frac{B_1 - A_1}{B_1 + A_1}$$

$$= \frac{rCs^3 + (2rC + C)s^2 + (rC + C)s + 1}{rCs^3 + (2rC + 2r + C)s^2 + (rC + C + 2r + 2)s + 1}$$

$$\equiv \frac{N_1(s)}{D_1(s)} \qquad (4)$$

Now, choose r and C such that the numerator polynomial $N_1(s)$ has $(s^2 + \omega^2)$ as a factor. The Routh array is applied to determine values of r and C. First of all, consider

$$N_1(s) = a_1 s^3 + b_1 s^2 + c_1 s + d_1 \qquad (5)$$

The Routh array formed from the even and odd polynomials of $N_1(s)$ is

$$\begin{array}{c|cc} s^3 & a_1 & c_1 \\ s^2 & b_1 & d_1 \\ s^1 & c_1 - \dfrac{a_1 d_1}{b_1} & 0 \end{array}$$

In order to have a common factor of the form $(s^2 + \omega^2)$, a row of zeros is required. The condition that elements of the third row be zero is

$$c_1 - \frac{a_1 d_1}{b_1} = 0 \tag{6}$$

Returning to the original numerator polynomial $N_1(s) = rCs^3 + (2rC + C)s^2 + (rC + C)s + 1$, and identifying with Eq. (5), one obtains

$$rC + C - \frac{rC \cdot 1}{2rC + C} = 0$$

or

$$C = \frac{r}{2r^2 + 3r + 1} \tag{7}$$

Equation (7) is one aquation involving two unknown parameters. Another equation, of the form $f(r) = 0$, is obtained by requiring that the value of C be a maximum. It is clear, without loss of generality, that the maximum value of C is desirable because of the low sensitivity of these two parametars in the neighborhood of a maximum.

In order to find the maximum C, one must calculate the first derivative of Eq. (7) with respect to r.

$$\frac{dC}{dr} = \frac{2r^2 + 3r + 1 - r(4r + 3)}{(2r^2 + 3r + 1)^2} \qquad (8)$$

Setting Eq. (8) equal to zero, one obtains

$$-2r^2 + 1 = 0$$

or

$$r = \frac{1}{\sqrt{2}} \text{ ohm} \qquad (9)$$

Substitution of $r = \frac{1}{\sqrt{2}}$ into Eq. (7) obtains

$$C = 3 - 2 \cdot \sqrt{2} = 0.17158 \text{ farad} \qquad (10)$$

The notch frequency can be determined by

$$b_1 s^2 + 1 = 0 \qquad (11)$$

That is,

$$(2rC + C)s^2 + 1 = 0$$

or

$$\omega^2 = \frac{1}{C(2r + 1)} \qquad (12)$$

Substitution of $C = \frac{r}{(r + 1)(2r + 1)}$ in Eq. (7) and $r = \frac{1}{\sqrt{2}}$ ohm in Eq. (9) into Eq. (12) yields

$$\omega^2 = \frac{r + 1}{r} = 2.4142$$

or

$$\omega = 1.5537 \text{ rad./sec.} \tag{13}$$

Construction of a Routh array from numarator and denominator polynomials yields:

The Routh Array

rC	2rC + 2r + C	rC + C + 2r + 2	1
rC	2rC + C	rC + C	0
2r	2r + 2	1	
rC	rC + C/2	0	
1	1		
C/2	0		
.			
.			
.			

There is no zero row, and hence there exists no common factors in the numerator and denominator polynomials.

Aftar substituting these specific values of r and C into Eq. (4), tha voltage transfer function bacomes

$$T_1(s) = \frac{0.121315\ s^3 + 0.41421\ s^2 + 0.292895\ s + 1}{0.121315\ s^3 + 1.82842\ \ s^2 + 3.707105\ s + 1}$$

In ordar to calculata the sinusoidal spectrum, let $s = j\omega$ and obtain

$$T_1(j\omega) = \frac{(1 - 0.41421\omega^2) + j(0.292895\omega - 0.121315\omega^3)}{(1 - 1.82842\omega^2) + j(3.707105\omega - 0.121315\omega^3)}$$

From this, one obtains

$$\left|T_1(j\omega)\right|^2 = \frac{(1 - 0.41421\omega^2)^2 + (0.292895\omega - 0.121315\omega^3)^2}{(1 - 1.82842\omega^2)^2 + (3.707105\omega - 0.121315\omega^3)^2}$$

By inspection, one can see that

$$\left|T_1(j\omega)\right|^2 = 1 \qquad \text{at} \quad \omega = 0$$

$$\left|T_1(j\omega)\right|^2 = 1 \qquad \text{at} \quad \omega = \infty$$

The log-log plot of $\left|T_1(j\omega)\right|^2$ versus ω is shown in Fig. 7. This frequency response shows that the transfer function $T_1(s)$ represents a notch network with equal amplitudes at zero and infinite frequencies. Data for this plot are obtained with Program 2 in Appendix C.

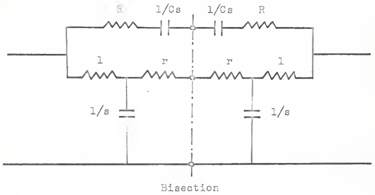

Fig. 1. A general notch filter of the Goldman type.

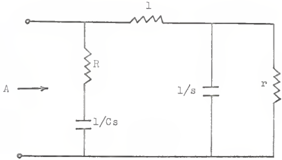

Fig. 2. Short-circuited half-section of the RC network in Fig. 1.

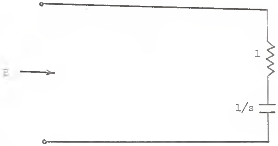

Fig. 3. Open-circuited half-section of the RC network in Fig. 1.

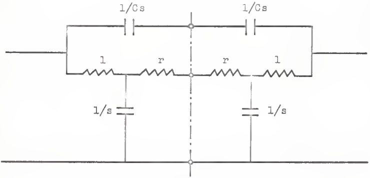

Fig. 4. Goldman type resistance-capacitance notch filter.

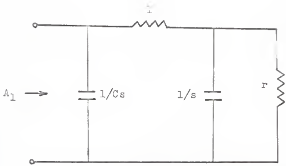

Fig. 5. Short-circuited half-section of the RC network in Fig. 4.

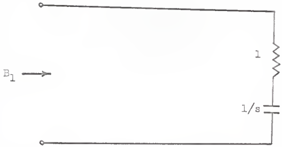

Fig. 6. Open-circuited half-section of the RC network in Fig. 4.

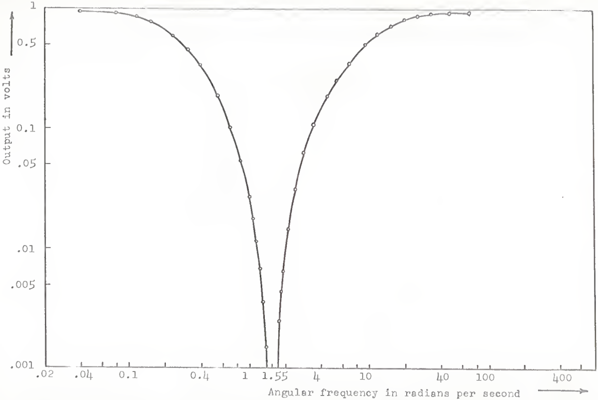

Fig. 7. Frequency response for the equal amplitudes notch network of the Goldman type.

UNEQUAL AMPLITUDES NOTCH FILTER

From the General Notch Filter, using the same networks shown in Figs. 1, 2, and 3, one obtains the transfer function as shown in Eq. (1), i.e.,

$$T_2(s) = \frac{B_2 - A_2}{B_2 + A_2}$$

$$= \frac{\left[rCs^3 + (2rC + C)s^2 + (RC + rC + C)s + 1\right]}{\left[(2RCr + rC)s^3 + (2RCr + 2RC + 2rC + 2r + C)s^2 + (RC + rC + C + 2r + 2)s + 1\right]}$$

$$\equiv \frac{N_2(s)}{D_2(s)} \tag{14}$$

The Routh array of the numerator polynomial's even and odd polynomials is

$$\begin{array}{c|ll} s^3 & rC & RC + rC + C \\ s^2 & 2rC + C & 1 \\ s^1 & RC + rC + C - \dfrac{rC}{2rC + C} & 0 \end{array}$$

In order to have the factor of $(s^2 + \omega^2)$, the third row should be equal to a zero row. Therefore

$$RC + rC + C - \frac{rC}{2rC + C} = 0$$

or

$$C = \frac{r}{(R + r + 1)(2r + 1)} \tag{15}$$

The notch frequency ω can be obtained from the second row of the Routh array

$$(2rC + C)s^2 + 1 = 0$$

or

$$\omega = \frac{1}{\sqrt{C(2r + 1)}} \tag{16}$$

Equation (16) shows that ω is independent of R.

Rearranging Eq. (15), one obtains

$$C = \frac{r}{2r^2 + (3 + 2R)r + (1 + R)} \tag{17}$$

Taking the first derivative of C with respect to r in Eq. (17) yields

$$\frac{dC}{dr} = \frac{\left[2r^2 + (3 + 2R)r + (1 + R)\right] - r\left[4r + (3 + 2R)\right]}{\left[2r^2 + (3 + 2R)r + (1 + R)\right]^2} \tag{18}$$

For the reason described previously, the maximum value of C is desirable. This is achieved by setting Eq. (18) equal to zero, namely,

$$2r^2 + (3 + 2R)r + (1 + R) - r\left[4r + (3 + 2R)\right] = 0$$

or

$$-2r^2 + (1 + R) = 0 \tag{19}$$

$$R = 2r^2 - 1 > 0 \tag{20}$$

Equation (20) implies that

$$r > \frac{\sqrt{2}}{2} \quad \text{ohm} \tag{21}$$

Substituting Eq. (20) into Eq. (17) yields

$$C = \frac{r}{2r^2 + (3 + 4r^2 - 2)r + 2r^2} = \frac{1}{4r^2 + 4r + 1} \tag{22}$$

Thus substitution of Eq. (22) into Eq. (16) yields

$$\begin{aligned} \omega &= \frac{1}{\sqrt{C(2r + 1)}} \\ &= \frac{1}{\sqrt{\dfrac{2r + 1}{4r^2 + 4r + 1}}} = \sqrt{2r + 1} \end{aligned} \tag{23}$$

First, assume $\omega = 1$ rad./sec. From Eq. (23) one obtains

$$r = \frac{1 - 1}{2} = 0 \quad \text{ohm} \tag{24}$$

Equation (24) contradicts the condition of Eq. (21).

Second, assume $\omega = 2$ rad./sec. Then one obtains from Eq. (23)

$$r = \frac{4 - 1}{2} = 1.5 \text{ ohms.}$$

This value does satisfy Eq. (21).

Substitution of $r = 1.5$ ohms into Eq. (20) and Eq. (22) yields respectively

$$R = 2r - 1 = 3.5 \text{ ohms}$$

$$C = \frac{1}{4r^2 + 4r + 1} = 0.0625 \text{ farad}$$

Returning to Eq. (14) for specific values of r, R and C, one obtains

$$T_2(s) = \frac{\left[rCs^3 + (2rC + C)s^2 + (RC + rC + C)s + 1\right]}{\left[(2RCr + rC)s^3 + (2RCr + 2RC + 2rC + 2r + C)s^2 + (RC + rC + C + 2r + 2)s + 1\right]}$$

$$= \frac{1.5\ s^3 + 4.0\ s^2 + 6.0\ s + 16}{12\ s^3 + 69.5\ s^2 + 86\ s + 16}$$

In order to calculate the sinusoidal spectrum, let $s = j\omega$ and obtain

$$T_2(j\omega) = \frac{(16 - 4.0\ \omega^2) + j(6.0\ \omega - 1.5\ \omega^3)}{(16 - 69.5\ \omega^2) + j(86\ \omega - 12\ \omega^3)}$$

From this, one obtains

$$\left|T_2(j\omega)\right|^2 = \frac{(16 - 4.0\ \omega^2)^2 + (6.0\ \omega - 1.5\ \omega^3)^2}{(16 - 69.5\ \omega^2)^2 + (86\ \omega - 12\ \omega^3)^2}$$

$$= \frac{2.25\ \omega^6 - 2\ \omega^4 - 92\ \omega^2 + 256}{144\ \omega^6 + 2766.25\ \omega^4 + 5172\ \omega^2 + 256}$$

By inspection, one can see that

$$\left|T_2(j\omega)\right|^2 = 1 \qquad \text{at} \quad \omega = 0$$

$$\left|T_2(j\omega)\right|^2 = 0.015625 \qquad \text{at} \quad \omega = \infty$$

A graph of $|T_2(j\omega)|^2$ versus ω on log-log paper is shown in Fig. 8. This frequency response shows that the transfer function $T_2(s)$ has a notch frequency and unequal amplitudes at zero and infinite frequencies. Data for this graph are obtained with Program 3 which is given in Appendix C.

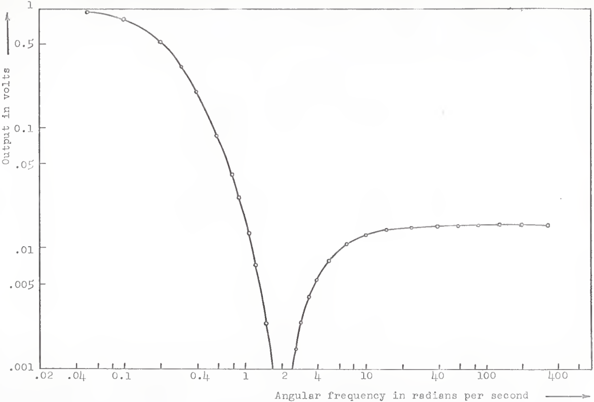

Fig. 8. Frequency response for the unequal amplitudes notch network.

AN EQUAL AMPLITUDES NOTCH FILTER

A general notch filter of equal amplitudes and unequal amplitudes has been investigated in the previous section. In this section, the more complicated notch filter resistance-capacitance network shown in Fig. 9 is investigated.

The mid short-circuit impedance and the mid open-circuit impedances are shown respectively in Fig. 10 and Fig. 11. Let A be the mid short-circuit input impedance; and let B be the mid open-circuit input impedance. After applying series and parallel operations to the network, one obtains

$$A = (1 + A_1) \left|\right| \frac{1}{Cs} \tag{25}$$

where

$$A_1 = \frac{(2m^2 + 4mr)s + (8m^2 + 8mr)}{(m^2 + 2mr)s^2 + (4mr + 4m^2 + 2m + 2r)s + 8m} \tag{26}$$

Therefore

$$A = \frac{\left[(m^2 + 2mr)s^2 + (6m^2 + 8mr + 2m + 2r)s + (8m^2 + 8m + 8mr)\right]}{\left[(m^2C + 2mrC)s^3 + (6m^2C + 8mrC + 2mC + 2rC + m^2 + 2mr)s^2 + (8m^2C + 8mC + 8mrC + 4mr + 4m^2 + 2m + 2r)s + 8m\right]}$$

$$\equiv \frac{N_a}{D_a} \tag{27}$$

Similarly,

$$B = 1 + B_1 \tag{28}$$

where

$$B_1 = \frac{2ms - 4m}{ms^2 + (2m + 1)s} \tag{29}$$

Therefore

$$B = \frac{ms^2 + (4m + 1)s + 4m}{ms^2 + (2m + 1)s} \equiv \frac{N_b}{D_b} \tag{30}$$

From Eq. (27) and Eq. (30), one obtains

$$\frac{B - A}{B + A} = \frac{N_b \cdot D_a - N_a \cdot D_b}{N_b \cdot D_a + N_a \cdot D_b} \tag{31}$$

where

$$\begin{aligned} N_b \cdot D_a = {} & (m^3C + 2m^2rC)s^5 + (10m^3C + 16m^2rC + 3m^2C \\ & + 4mrC + m^3 + 2m^2r)s^4 + (36m^3C + 22m^2C + 2mC \\ & + 48m^2rC + 16mrC + 2rC + 8m^3 + 3m^2 + 12m^2r \\ & + 4mr)s^3 + (56m^3C + 48m^2C + 8mC + 64m^2rC \\ & + 16mrC + 20m^3 + 20m^2 + 24m^2r + 12mr + 2m \\ & + 2r)s^2 + (32m^3C + 32m^2C + 32m^2rC + 16m^3 \\ & + 40m^2 + 16m^2r + 8mr + 8m)s + 32m^2 \end{aligned} \tag{32}$$

$$\begin{aligned} N_a \cdot D_b = {} & (m^3 + 2m^2r)s^4 + (8m^3 + 3m^2 + 12m^2r + 4mr)s^3 \\ & + (20m^3 + 18m^2 + 24m^2r + 12mr + 2m + 2r)s^2 \\ & + (16m^3 + 24m^2 + 16m^2r + 8mr + 8m)s \end{aligned} \tag{33}$$

Bartlett's representation theorem for a symmetric network yields the transfer function

$$T(s) = \frac{B - A}{B + A} = \frac{N_b \cdot D_a - N_a \cdot D_b}{N_b \cdot D_a + N_a \cdot D_b} \equiv \frac{N(s)}{D(s)} \tag{34}$$

where

$$
\begin{aligned}
N(s) = {} & (m^3C + 2m^2rC)s^5 + (10m^3C + 16m^2rC + 3m^2C \\
& + 4mrC)s^4 + (36m^3C + 22m^2C + 48m^2rC + 16mrC \\
& + 2mC + 2rC)s^3 + (56m^3C + 48m^2C + 64m^2rC + 8mC \\
& + 16mrC + 2m^2)s^2 + (32m^3C + 32m^2C + 32m^2rC \\
& + 16m^2)s + 32m^2
\end{aligned} \tag{35}
$$

$$
\begin{aligned}
D(s) = {} & (m^3C + 2m^2rC)s^5 + (10m^3C + 16m^2rC + 3m^2C + 4mr \\
& + 2m^3 + 4m^2r)s^4 + (36m^3C + 22m^2C + 48m^2rC \\
& + 24m^2r + 16m^3 + 6m^2 + 8mr + 16mrC + 2mC \\
& + 2rC)s^3 + (56m^3C + 48m^2C + 64m^2rC + 8mC + 16mrC \\
& + 40m^3 + 38m^2 + 48m^2r + 24mr + 4m + 4r)s^2 \\
& + (32m^3C + 32m^2C + 32m^2rC + 32m^3 + 64m^2 + 32m^2r \\
& + 16mr + 16m)s + 32m^2
\end{aligned} \tag{36}
$$

In order for this transfer function $T(s)$ to have a notch in its frequency response, the values of m, r, and C must be so chosen that the numerator polynomial $N(s)$ has a factor of the form $(s^2 + \omega^2)$. The values of m, r, and C can be determined by dividing $N(s)$ into even and odd polynomials which are subjected to a Routh array calculation. First of all, let $N(s)$ be denoted as:

$$
N(s) = a_5s^5 + a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a_0 \tag{37}
$$

The Routh array yields

$$
\begin{array}{c|ccc}
s^5 & a_5 & a_3 & a_1 \\
s^4 & a_4 & a_2 & a_0 \\
s^3 & \dfrac{a_3a_4 - a_2a_5}{a_4} = P_1 & \dfrac{a_1a_4 - a_0a_5}{a_4} = Q_1 & 0 \\
s^2 & \dfrac{P_1a_2 - Q_1a_4}{P_1} = R_1 & a_0 & 0 \\
s^1 & \dfrac{R_1Q_1 - P_1a_0}{R_1} & 0 & 0
\end{array}
$$

There must be a row of zeros at the end of this Routh array in order to find a common factor in these polynomials, namely,

$$Q_1 = \frac{P_1}{R_1} \cdot a_0 \tag{38}$$

Substitution of these identical representations of P_1, Q_1, and R_1 into Eq. (38) yields

$$
\begin{aligned}
&(a_3a_4 - a_2a_5)\left[(a_1a_4 - a_0a_5)a_2 - (a_3a_4 - a_2a_5)a_0\right] \\
&= (a_1a_4 - a_0a_5)^2 \cdot a_4
\end{aligned} \tag{39}
$$

The other factor which determines the notch frequency ω is

$$(a_2 - \frac{Q_1}{P_1} \cdot a_4)s^2 + a_0 = 0 \tag{40}$$

or

$$
\begin{aligned}
&\left[a_2(a_3a_4 - a_2a_5) - a_4(a_1a_4 - a_0a_5)\right]s^2 \\
&+ a_0(a_3a_4 - a_2a_5) = 0
\end{aligned} \tag{41}
$$

On comparing coefficients of polynomials in Eqs. (35) and (37), one identifies

$$a_5 = m^3C + 2m^2rC$$
$$a_4 = 10m^3C + 3m^2C + 16m^2rC + 4mrC$$
$$a_3 = 36m^3C + 22m^2C + 48m^2rC + 16mrC + 2mC + 2rC$$
$$a_2 = 56m^3C + 48m^2C + 64m^2rC + 16mrC + 8mC + 2m^2$$
$$a_1 = 32m^3C + 32m^2C + 32m^2rC + 16m^2$$
$$a_0 = 32m^2$$

In order to achieve Eq. (39), several steps are calculated separately as follows:

$$\begin{aligned}
&(a_1a_4 - a_0a_5) \cdot a_2 \\
&= \Big[(32m^3C + 32m^2C + 32m^2rC + 16m^2) \cdot (10m^3C + 3m^2C + 16m^2rC \\
&\quad + 4mrC) - 32m^2 \cdot (m^3C + 2m^2rC)\Big] \cdot \Big[(56m^3C + 48m^2C + 64m^2rC \\
&\quad + 16mrC + 8mC + 2m^2)\Big]
\end{aligned}$$

$$\begin{aligned}
&= 32m^3C \cdot \Big[(560C^2m^6 + 1208C^2m^5 + 2096C^2m^5r + 872C^2m^4 \\
&\quad + 3528C^2m^4r + 2560C^2m^4r^2 + 248C^2m^3 + 1936C^2m^3r \\
&\quad + 2880C^2m^3r^2 + 1024C^2m^3r^3 + 24C^2m^2 + 424C^2m^2r + 944C^2m^2r^2 \\
&\quad + 512C^2m^2r^3 + 32C^2mr + 64C^2mr^2 + 64C^2mr^3) + (244Cm^5 \\
&\quad + 302Cm^4 + 644Cm^4r + 110Cm^3 + 606Cm^3r + 416Cm^3r^2 + 12Cm^2 \\
&\quad + 176Cm^2r + 232Cm^2r^2 + 16Cmr + 32Cmr^2 + 32Cmr^3) \\
&\quad + (8m^4 + 3m^3 + 12m^3r + 4m^2r)\Big]
\end{aligned} \tag{42}$$

$$\begin{aligned}
&(a_3a_4 - a_2a_5) \cdot a_0 \\
&= \Big[(36m^3C + 22m^2C + 48m^2rC + 16mrC + 2mC + 2rC) \cdot (10m^3C \\
&\quad + 3m^2C + 16m^2rC + 4mrC) - (56m^3C + 48m^2C + 64m^2rC + 16mrC \\
&\quad + 8mC + 2m^2) \cdot (m^3C + 2m^2rC)\Big] \cdot 32m^2
\end{aligned}$$

$$= 32m^3C\ (304Cm^5 + 280Cm^4 + 880Cm^4r + 688Cm^3r + 78Cm^3$$

$$+ 640Cm^3r^2 + 6Cm^2 + 172Cm^2r + 416Cm^2r^2 + 14Cmr + 96Cmr^2$$
$$+ 8Cr^2 - 2m^4 - 4m^3r) \qquad (43)$$

$$(a_1a_4 - a_0a_5)a_2 - (a_3a_4 - a_2a_5)a_0$$
$$= 32m^3C\,[(560C^2m^6 + 1208C^2m^5 + 2096C^2m^5r + 872C^2m^4 + 3528C^2m^4r$$
$$+ 2560C^2m^4r^2 + 248C^2m^3 + 1936C^2m^3r + 2880C^2m^3r^2 + 1024C^2m^3r^3$$
$$+ 24C^2m^2 + 424C^2m^2r + 944C^2m^2r^2 + 512C^2m^2r^3 + 32C^2mr$$
$$+ 64C^2mr^2 + 64C^2mr^3) + (-60Cm^5 + 22Cm^4 - 236Cm^4r + 32Cm^3$$
$$- 82Cm^3r - 224Cm^3r^2 + 6Cm^2 + 4Cm^2r - 184Cm^2r^2 + 24Cmr$$
$$- 64Cmr^2 + 32Cmr^3 - 8Cr^2) + (10m^4 + 3m^2 + 16m^3r$$
$$+ 4m^2r)] \qquad (44)$$

$$(a_1a_4 - a_0a_5)^2 \cdot a_4$$
$$= 64m^4C^2 \cdot (20Cm^4 + 26Cm^3 + 52Cm^3r + 6Cm^2 + 46Cm^2r$$
$$+ 32Cm^2r^2 + 8Cmr + 8Cmr^2 + 8m^3 + 3m^2 + 12m^2r + 4mr)^2$$
$$\cdot (40Cm^3 + 64Cm^2r + 12Cm^2 + 16Cmr) \qquad (45)$$

Multiplying Eq. (44) by $(a_3a_4 - a_2a_5)$ and expanding it, then combining with Eq. (45), one obtains a polynomial equation $H(m, r, C)$ identical to Eq. (39) such that

$$H(m, r, C) \equiv \alpha C^3 + \beta C^2 + \gamma C + \delta = 0 \qquad (46)$$

where

$$\alpha = 69120m^{11} + 215616m^{10} + 456192m^{10}r + 274383m^9$$
$$+ 1274176m^9r + 1198080m^9r^2 + 185096m^8 + 1454096m^8r$$
$$+ 2929920m^8r^2 + 1564672m^8r^3 + 70816m^7 + 882888m^7r$$
$$+ 2932096m^7r^2 + 3246080m^7r^3 + 1051808m^7r^4 + 15216m^6$$
$$+ 306880m^6r + 1559888m^6r^2 + 2758144m^6r^3 + 1703936m^6r^4$$

$$+ 262144m^6r^5 + 1680m^5 + 60784m^5r + 477320m^5r^2 + 1233280m^5r^3$$
$$+ 1167360m^5r^4 + 327680m^5r^5 + 72m^4 + 6320m^4r + 84432m^4r^2$$
$$+ 313920m^4r^3 + 409600m^4r^4 + 163840m^4r^5 + 264m^3r + 8056m^3r^2$$
$$+ 46032m^3r^3 + 77184m^3r^4 + 40960m^3r^5 + 320m^2r^2 + 3680m^2r^3$$
$$+ 7296m^2r^4 + 5120m^2r^5 + 128mr^3 + 256mr^4 + 256mr^5. \qquad (47)$$

$$\beta = -22480m^{10} - 31544m^9 - 138448m^9r - 11780m^8 - 210216m^8r$$
$$- 318208m^8r^2 + 1358m^7 - 84196m^7r - 424928m^7r^2 - 323584m^7r^3$$
$$+ 1698m^6 - 8090m^6r - 207944m^6r^2 - 407808m^6r^3 - 12880m^6r^4$$
$$+ 330m^5 + 3352m^5r - 44088m^5r^2 - 206208m^5r^3 - 133376m^5r^4$$
$$+ 18m^4 + 830m^4r - 3206m^4r^2 - 59808m^4r^3 - 69888m^4r^4$$
$$+ 1024m^4r^5 + 48m^3r + 112m^3r^2 - 15568m^3r^3 - 23872m^3r^4$$
$$+ 6656m^3r^5 + 14m^2r^2 - 1024m^2r^3 - 5248m^2r^4 + 1536m^2r^5$$
$$- 48mr^3 - 640mr^4 + 128mr^5 - 32r^4 \qquad (48)$$

$$\gamma = -980m^9 - 854m^8 - 4588m^8r - 158m^7 - 4194m^7r - 7112m^7r^2$$
$$+ 33m^6 - 928m^6r - 6692m^6r^2 - 3648m^6r^3 + 9m^5 - 54m^5r$$
$$- 2552m^5r^2 - 3504m^5r^3 + 33m^4r + 68m^4r^2 - 832m^4r^3 - 64m^4r^4$$
$$+ 40m^3r^2 + 16m^3r^3 + 16m^2r \qquad (49)$$

$$\delta = -10m^8 - 3m^7 - 36m^7r - 10m^6r - 32m^6r^2 - 8m^5r^2 \qquad (50)$$

Returning to Eq. (41), one identifies

$$a_2(a_3a_4 - a_2a_5)$$
$$= \left[(56m^3c + 48m^2c + 64m^2rc + 16mrc + 8mc + 2m^2)\right]$$
$$\cdot \left[2mc(152m^5c + 140m^4c + 440m^4rc + 344m^3rc + 39m^3c\right.$$
$$+ 320m^3r^2c + 3m^2c + 86m^2rc + 208m^2r^2c + 7mrc + 48mr^2c$$
$$\left. + 4r^2c - m^4 - 2m^3r)\right] \qquad (51)$$

$$a_4(a_1a_4 - a_0a_5)$$
$$= 2mC\cdot(160m^5C + 208m^4C + 416m^4rC + 48m^3C + 368m^3rC$$
$$+ 256m^3r^2C + 64m^2rC + 64m^2r^2C + 64m^4 + 96m^3r + 24m^3$$
$$+ 32m^2r) \cdot (10m^3C + 16m^2rC + 3m^2C + 4mrC) \quad (52)$$

$$a_0(a_3a_4 - a_2a_5)$$
$$= 4m^2C \cdot (2432m^6C + 2240m^5C + 7040m^5rC + 5504m^4rC + 624m^4C$$
$$+ 5120m^4r^2C + 48m^3C + 1376m^3rC + 3328m^3r^2C + 112m^2rC$$
$$+ 768m^2r^2C + 64mr^2C - 16m^5 - 32m^4r) \quad (53)$$

Substituting Eqs. (51), (52), and (53) into Eq. (41) and expanding them term by term, one obtains ω^2.

$$\omega^2 = \frac{p_1C + q_1}{p_2C^2 + t_2C + q_2} \quad (54)$$

where

$$p_1 = 2432m^6 + 2240m^5 + 7040m^4r + 5504m^4r + 624m^4 + 5120m^4r^2$$
$$+ 48m^3 + 1376m^3r + 3328m^3r^2 + 112m^2r + 768m^2r^2 + 64mr^2 \quad (55)$$

$$q_1 = -16m^5 - 32m^4r \quad (56)$$

$$p_2 = 3456m^7 + 6288m^6 + 13824m^6r + 4508m^5 + 21440m^5r$$
$$+ 18432m^5r^2 + 1508m^4 + 13120m^4r + 23552m^4r^2 + 8192m^4r^3$$
$$+ 228m^3 + 3852m^3r + 11776m^3r^2 + 8192m^3r^3 + 12m^2 + 536m^2r$$
$$+ 2880m^2r^2 + 3072m^2r^3 + 28mr + 344mr^2 + 512mr^3 + 16r^2$$
$$+ 32r^3 \quad (57)$$

$$t_2 = -196m^6 - 100m^5 - 648m^5r - m^4 - 336m^4r - 512m^4r^2 + 3m^3$$
$$- 18m^3r - 256m^3r^2 + 7m^2r - 16m^2r^2 + 4mr^2 \quad (58)$$

$$q_2 = -m^5 - 2m^4 r \tag{59}$$

By the ergument used previously, one must heve the meximum velue of C in Eq. (46). It is cleer thet one cen take the per-tiel derivetive of H(m, r, C) with respect to m end obtein

$$\frac{\partial H}{\partial m} = 3\alpha C^2 \frac{\partial C}{\partial m} + C^3 \frac{\partial \alpha}{\partial m} + 2\beta C \frac{\partial C}{\partial m} + C^2 \frac{\partial \beta}{\partial m} + \gamma \frac{\partial C}{\partial m}$$
$$+ C \frac{\partial \gamma}{\partial m} + \frac{\partial \delta}{\partial m} = 0 \tag{60}$$

Putting $\frac{\partial C}{\partial m} = 0$ in Eq. (60) yields

$$\frac{\partial \alpha}{\partial m} C^3 + \frac{\partial \beta}{\partial m} C^2 + \frac{\partial \gamma}{\partial m} C + \frac{\partial \delta}{\partial m} = 0 \tag{61}$$

In order to simplify the notetions, some symbole used in the following peges ere:

$$\alpha_1 = \frac{\partial \alpha}{\partial m} \qquad \alpha_2 = \frac{\partial \alpha}{\partial r}$$

$$\alpha_{11} = \frac{\partial^2 \alpha}{\partial m^2} \qquad \alpha_{22} = \frac{\partial^2 \alpha}{\partial r^2}$$

$$\alpha_{12} = \frac{\partial^2 \alpha}{\partial m \partial r} \qquad C_1 = \frac{\partial C}{\partial m}$$

$$\beta_1 = \frac{\partial \beta}{\partial m} \qquad \beta_2 = \frac{\partial \beta}{\partial r}$$

$$\beta_{11} = \frac{\partial^2 \beta}{\partial m^2} \qquad \beta_{22} = \frac{\partial^2 \beta}{\partial r^2}$$

$$\beta_{12} = \frac{\partial^2 \beta}{\partial m \partial r} \qquad C_2 = \frac{\partial C}{\partial r}$$

$$\gamma_1 = \frac{\partial \gamma}{\partial m} \qquad \gamma_2 = \frac{\partial \gamma}{\partial r}$$

$$\gamma_{11} = \frac{\partial^2 \gamma}{\partial m^2} \qquad \gamma_{22} = \frac{\partial^2 \gamma}{\partial r^2}$$

$$\gamma_{12} = \frac{\partial^2 \gamma}{\partial m \partial r} \qquad C_{12} = \frac{\partial^2 C}{\partial m \partial r}$$

$$\delta_1 = \frac{\partial \delta}{\partial m} \qquad \delta_2 = \frac{\partial \delta}{\partial r}$$

$$\delta_{11} = \frac{\partial^2 \delta}{\partial m^2} \qquad \delta_{22} = \frac{\partial^2 \delta}{\partial r^2}$$

$$\delta_{12} = \frac{\partial^2 \delta}{\partial m \partial r} \qquad C_{11} = \frac{\partial^2 C}{\partial m^2}$$

Returning to Eq. (46), $H(m, r, C) \equiv \alpha C^3 + \beta C^2 + \gamma C + \delta = 0$, one can see that this is a cubic equation of C in terms of m and r. Generally, a cubic equation can be solved using Carden's formulas (11) which are described in the literature. However, all coefficients of C in Eq. (46), obviously, are complicated polynomials in terms of m, r. Thus it would be difficult to solve for C in terms of m, r by the classical method. Also, since it is desirable to use the computer for calculation, the complexity of the expression for C would lead to problems with computer storage capacity.

To avoid these problems, an optimum process based on the gradient vector method (Ref. 9) is applied to solve for the specific values of m, r, and C in Eq. (46), in which C is a maximum value. This method is described below.

Rearranging Eq. (46) yields

$$C^2 = \frac{-(\gamma C + \delta)}{\alpha C + \beta} \tag{62}$$

In order to obtain the maximum value of C, one must take the first and second partial derivative of C with respect to m. Then putting $C_1 = C_{11} = 0$, yields a linear form of C in terms of m, r. Several steps are achieved as follows. First of all, taking the partial derivative of C in Eq. (62) with respect to m yields

$$2CC_1 = \frac{(\alpha C + \beta)(-\gamma_1 C - \gamma C_1 - \delta_1) + (\gamma C + \delta)(\alpha C_1 + \alpha_1 C + \beta_1)}{(\alpha C + \beta)^2} \tag{63}$$

For convenience in calculation, one puts $C_1 = 0$ in Eq. (63) before proceeding to calculate C_{11}. Therefore, Eq. (63) becomes

$$(\alpha C + \beta)(-\gamma_1 C - \delta_1) + (\gamma C + \delta)(\alpha_1 C + \beta_1) = 0 \tag{64}$$

Rearranging Eq. (64) yields

$$C^2 = \frac{\alpha_1 \delta C + \beta_1 \gamma C - \beta \gamma_1 C - \alpha \delta_1 C - \beta \delta_1 + \beta_1 \delta}{\alpha \gamma_1 - \alpha_1 \gamma} \tag{65}$$

Secondly, taking the partial derivative of C in Eq. (65) with respect to m yields

$$2CC_1 = \frac{\begin{aligned}&[(\alpha\gamma_1 - \alpha_1\gamma)(\alpha_{11}\delta C + \alpha_1\delta C_1 + \beta_{11}\gamma C + \beta_1\gamma C_1 - \beta\gamma_{11}C \\ &- \beta\gamma_1 C_1 - \alpha\delta_{11}C - \beta\delta_{11} + \beta_{11}\delta) - (\alpha_1\delta C + \beta_1\gamma C \\ &- \beta\gamma_1 C - \alpha\delta_1 C - \beta\delta_1 + \beta_1\delta)(\alpha\gamma_{11} - \alpha_{11}\gamma)\end{aligned}}{(\alpha\gamma_1 - \alpha_1\gamma)^2} \tag{66}$$

Simplifying Eq. (66) and putting $C_1 = 0$ again, one obtains

$$C = \frac{\begin{bmatrix}\alpha\beta\gamma_{11}\delta_1 - \alpha\beta\gamma_1\delta_{11} + \alpha\beta_{11}\gamma_1\delta - \alpha_{11}\beta\gamma\delta_1 + \alpha_1\beta\gamma\delta_1 \\ - \alpha\beta_1\gamma_{11}\delta + \alpha_{11}\beta_1\gamma\delta - \alpha_1\beta_{11}\gamma\delta\end{bmatrix}}{\begin{bmatrix}\alpha^2\gamma_1\delta_{11} - \alpha^2\gamma_{11}\delta_1 + \alpha\alpha_{11}\gamma\delta_1 - \alpha\alpha_{11}\gamma_1\delta + \alpha_{11}\beta\gamma\gamma_1 \\ - \alpha\beta_{11}\gamma\gamma_1 + \alpha_1\beta_{11}\gamma^2 - \alpha_{11}\beta_1\gamma^2 + \alpha\beta_1\gamma\gamma_{11} - \alpha_1\beta\gamma\gamma_{11}\end{bmatrix}}$$

$$\equiv \frac{A + B}{D + E} \tag{67}$$

Similarly, taking the derivative of Eq. (62) with respect to r and using the procedures described above yields

$$C = \frac{\begin{bmatrix}\alpha\beta\gamma_{22}\delta_2 - \alpha\beta\gamma_2\delta_{22} + \alpha\beta_{22}\gamma_2\delta - \alpha_{22}\beta\gamma\delta_2 + \alpha_2\beta\gamma\delta_2 \\ - \alpha\beta_2\gamma_{22}\delta + \alpha_{22}\beta_2\gamma\delta - \alpha_2\beta_{22}\gamma\delta\end{bmatrix}}{\begin{bmatrix}\alpha^2\gamma_2\delta_{22} - \alpha^2\gamma_{22}\delta_2 + \alpha\alpha_{22}\gamma\delta_2 - \alpha\alpha_{22}\gamma_2\delta + \alpha_{22}\beta\gamma\gamma_2 \\ - \alpha\beta_{22}\gamma\gamma_2 + \alpha_2\beta_{22}\gamma^2 - \alpha_{22}\beta_2\gamma^2 + \alpha\beta_2\gamma\gamma_{22} - \alpha_2\beta\gamma\gamma_{22}\end{bmatrix}}$$

$$\equiv \frac{U + V}{P + Q} \tag{68}$$

Equating Eq. (67) and Eq. (68), one obtains a polynomial equation G(m, r) in terms of m and r, i.e.,

$$G(m, r) \equiv \frac{A + B}{D + E} - \frac{U + V}{P + Q} = 0 \tag{69}$$

Substitution of $C = \frac{A + B}{D + E}$ into Eq. (61) yields

$$H(m, r) \equiv \alpha_1\left(\frac{A + B}{D + E}\right)^3 + \beta_1\left(\frac{A + B}{D + E}\right)^2 + \gamma_1\left(\frac{A + B}{D + E}\right) + \delta_1 = 0 \tag{70}$$

The value of C_{12} (i.e., $\frac{\partial^2 C}{\partial m \partial r}$) shall be investigated to assure that C has a relative maximum. If $C_{12} = 0$, then C has a saddle point. If $C_{12} \neq 0$ (greater or less than zero), then C has a relative maximum or minimum.

Returning to Eq. (63),

$$2CC_1 = \frac{(\alpha C + \beta)(-\gamma_1 C - \gamma C_1 - \delta_1) + (\gamma C + \delta)(\alpha C_1 + \alpha_1 C + \beta_1)}{(\alpha C + \beta)^2} \tag{71}$$

Taking the partial derivative of Eq. (71) with respect to r yields

$$\begin{aligned} 2(CC_{12} - C_1 C_2) = {} & \frac{\frac{\partial}{\partial r}\left[(\alpha C + \beta)(-\gamma_1 C - \gamma C_1 - \delta_1)\right]}{(\alpha C + \beta)^2} \\ & - \frac{(\alpha C + \beta)(-\gamma_1 C - \gamma C_1 - \delta_1) \cdot \frac{\partial}{\partial r}(\alpha C + \beta)^2}{(\alpha C + \beta)^4} \\ & + \frac{\frac{\partial}{\partial r}\left[(\gamma C + \delta)(\alpha C_1 + \alpha_1 C + \beta_1)\right]}{(\alpha C + \beta)^2} \\ & - \frac{(\gamma C + \delta)(\alpha C_1 + \alpha_1 C + \beta_1) \cdot \frac{\partial}{\partial r}(\alpha C + \beta)^2}{(\alpha C + \beta)^4} \end{aligned} \tag{72}$$

Expanding and rearranging Eq. (72), one obtains

$$2CC_{12} - \frac{\alpha\delta - \beta\gamma}{(\alpha C + \beta)^2} \cdot C_{12}$$

$$= \frac{1}{(\alpha C + \beta)^2} \Big[C^2(\alpha_{12}\gamma - \alpha\gamma_{12} + \alpha_1\gamma_2 + \alpha_2\gamma_1) + C(\alpha_1\delta_2 + \alpha_2\delta_1$$
$$+ \beta_2\gamma_1 + \beta_1\gamma_2 + \alpha_{12}\delta - \alpha\delta_{12} + \beta_{12}\gamma - \beta\gamma_{12}) + (\beta_1\delta_2 + \beta_2\delta_1$$
$$+ \beta_{12}\delta - \beta\delta_{12}) \Big] - \frac{2}{(\alpha C + \beta)^3} (\gamma C + \delta)(\alpha_1 C + \beta_1)(\alpha_2 C + \beta_2). \tag{73}$$

Thus

$$C_{12} = \frac{1}{2C(\alpha C + \beta)^2 - \alpha\delta + \beta\gamma} \Bigg\{ \Big[C^2(\alpha_{12}\gamma - \alpha\gamma_{12} + \alpha_1\gamma_2 + \alpha_2\gamma_1)$$
$$+ C(\alpha_1\delta_2 + \alpha_2\delta_1 + \beta_2\gamma_1 + \beta_1\gamma_2 + \alpha_{12}\delta - \alpha\delta_{12} + \beta_{12}\gamma$$
$$- \beta\gamma_{12}) + (\beta_1\delta_2 + \beta_2\delta_1 + \beta_{12}\delta - \beta\delta_{12}) \Big]$$
$$- \left[\frac{2}{\alpha C + \beta} (\gamma C + \delta)(\alpha_1 C + \beta_1)(\alpha_2 C + \beta_2) \right] \Bigg\}$$
$$\equiv \frac{WX - WY}{WZ} \tag{74}$$

Among those solutions of C, m, r, C_{12}, in Program 1, one educated guess is

$C = 9185.78979 \times 10^{-6}$

$\simeq 9185.79 \times 10^{-6}$ farad

$m = 0.3042$

$r = 0.0179$ ohm

$C_{12} = 67.431$ farad per ohm.

For this set of solutions, C_{12} is greater than zero, so the dependent C possesses a maximum.

Substitution of these values of C, m, and r into Eq. (34) yields

$$T(s) = \frac{N(s)}{D(s)}$$

where

$$N(s) = (0.03053742s^5 + 0.5999928s^4 + 3.8378482s^3 + 28.7633238s^2 + 165.0841871s + 322.3679696) \times C$$

$$D(s) = (0.03053742s^5 + 7.4503121s^4 + 122.3840641s^3 + 677.1501976s^2 + 1291.8172755s + 322.3679696) \times C$$

Furthermore, substituting $s = j\omega$ yields

$$T(j\omega) = \frac{\left[(0.5999928\omega^4 - 28.7633238\omega^2 + 322.3679696) + j(0.03053742\omega^5 - 3.8378482\omega^3 + 165.0841871\omega)\right]}{\left[(7.4503121\omega^4 - 677.1501976\omega^2 + 322.3679696) + j(0.03053742\omega^5 - 122.3840641\omega^3 + 1291.8172755\omega)\right]}$$

$$\left|T(j\omega)\right|^2 = \frac{\left[(0.5999928\omega^4 - 28.7633238\omega^2 + 322.3679696)^2 + (0.03053742\omega^5 - 3.8378482\omega^3 + 165.0841871\omega)^2\right]}{\left[(7.4503121\omega^4 - 677.1501976\omega^2 + 322.3679696)^2 + (0.03053742\omega^5 - 122.3840641\omega^3 + 1291.8172755\omega)^2\right]}$$

By inspection, one can see that

$$\left|T(j\omega)\right|^2 = 1 \qquad \text{at } \omega = 0$$

$$\left|T(j\omega)\right|^2 = 1 \qquad \text{at } \omega = \infty$$

The log-log plot of $\left|T(j\omega)\right|^2$ versus ω is shown in Fig. 12. This frequency response shows that the transfer function $T(s)$ represents a notch network with equal amplitudes at zero and infinite frequencies. Data for this plot are obtained with Program 4 in Appendix C.

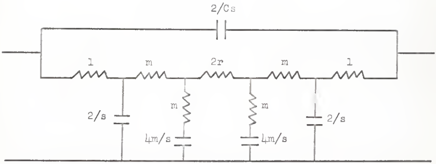

Fig. 9. An equal amplitude notch filter resistance-capacitance network.

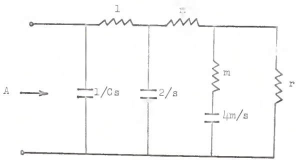

Fig. 10. Mid short-circuited impedance in Fig. 9.

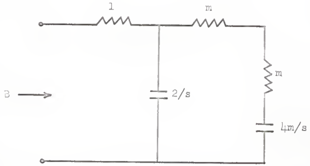

Fig. 11. Mid open-circuited impedance in Fig. 9.

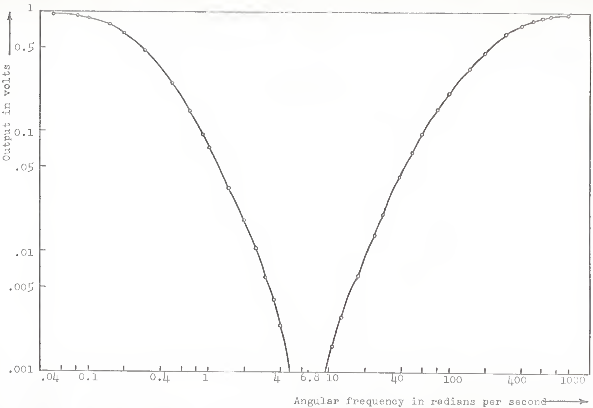

Fig. 12. Frequency response for the equal amplitudes notch network.

COMPARISON OF NOTCH FILTER Q'S

The parameter Q represents the sherpness of the circuit frequency response curve. It can be defined as

$$Q = 2\pi \cdot \frac{\text{energy stored in circuit}}{\text{energy dissipeted in circuit during one cycle}} .$$

Similarly, the fector Q defines the sharpness of the notch spectrum. The bendwidth of e notch filter is commonly defined es the width of the bend of frequencies over which the output power does not drop to less than one-helf, or -3 db of the output power at notch frequency. The explicit formule is

$$Q = \frac{f_0}{(\Delta f_0)_{3db}} = \frac{\omega_0}{(\Delta \omega_0)_{3db}}$$

where f_0 is the notch frequency end Δf_0 is the number of cycles off notch frequency et which the responses are 70.7 per cent of its peak value. In other words, this factor Q is the reciprocal of the bandwidth in units of the notch frequency.

Case 1. Equel amplitudes notch filter of the Goldmen type.

From Fig. 4, which is the response for equal amplitudes notch filter of the Goldman type, one obtains

$$Q_1 = \frac{1.55}{19.55 - 0.192} = 0.0803$$

Case 2. Unequal amplitudes notch filter.

The bandwidth is undefined because of the unequal magnitudes of the curve (Fig. 8) at zero and infinite frequencies. Therefore Q_2 for this filter is undeterminable.

Case 3. Equal amplitudes notch filter.

$$Q_3 = \frac{6.82}{352.3 - 0.18} = 0.0193$$

Q_3 is smaller than Q_1; the notch for the simple Goldman type network is sharper than that of the complicated one. The smaller Q_3 for the complicated network indicates that a wider bandwidth is achieved; the bandwidth is increased by 76 per cent. Increased bandwidth is desirable in carrier-frequency servomechanisms subjected to wide bandwidth input signals.

CONCLUSION

An RC notch filter has a zero minimum frequency response curve. Three cases have been exhibited in this paper:

1. An equal amplitude RC network of the Goldman type.
2. An unequel amplitude RC network.
3. An equal amplitude RC network (see Fig. 9).

Determination of proper paremeters for the existence of a notch frequency in eech case is emphasized. Particularly in the third case, an optimum process is applied to decide the specific velues of m, r, end C for the notch frequency to occur at 6.82 redians per second. (See Progrem 1.)

The graph for the frequency response in each case shows its particular amplitude characteristic.

Generalized Goldman RC notch networks have led to a meximum problem of an unorthodox type. Specific procedures have been exhibited for reducing this meximum problem to the problem of solving two bivariate polynomials. Numericel trial end error procedures affecting numerical solutions on a digital computer are given in Appendix C.

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APPENDICES

APPENDIX A

Description of the IBM-1620 Computer Program Used for Solving Two Bivariate Polynomial Equations in Program 1.

Several numerical methods are available for solving non-linear polynomial equations. The procedure selected was the numerical trial and error method.

From Eqs. (69), (70) in this paper, one obtains

$$G(m,r) \equiv \frac{A + B}{D + E} - \frac{U + V}{P + Q} = 0 \tag{75}$$

$$H(m,r) \equiv \alpha_1\left(\frac{A + B}{D + E}\right)^3 + \beta_1\left(\frac{A + B}{D + E}\right)^2 + \gamma_1\left(\frac{A + B}{D + E}\right) + \delta_1 = 0 \tag{76}$$

where all parameters in Eqs. (75) and (76) are high power polynomials in terms of m, r.

First of all, let some symbols used in Program 1 be denoted as:

$AX = \alpha$	$CX = \gamma$
$AM1 = \alpha_1 = \frac{\partial\alpha}{\partial m}$	$CM1 = \gamma_1 = \frac{\partial\gamma}{\partial m}$
$AM2 = \alpha_{11} = \frac{\partial^2\alpha}{\partial m^2}$	$CM2 = \gamma_{11} = \frac{\partial^2\gamma}{\partial m^2}$
$AM1R1 = \alpha_{12} = \frac{\partial^2\alpha}{\partial m\partial r}$	$CM1R1 = \gamma_{12} = \frac{\partial^2\gamma}{\partial m\partial r}$

$$AR1 = \alpha_2 = \frac{\partial \alpha}{\partial r} \qquad CR1 = \gamma_2 = \frac{\partial^2 \gamma}{\partial r}$$

$$AR2 = \alpha_{22} = \frac{\partial^2 \alpha}{\partial r^2} \qquad CR2 = \gamma_{22} = \frac{\partial^2 \gamma}{\partial r^2}$$

$$BX = \beta \qquad DX = \delta$$

$$BM1 = \beta_1 = \frac{\partial \beta}{\partial m} \qquad DM1 = \delta_1 = \frac{\partial \delta}{\partial m}$$

$$BM2 = \beta_{11} = \frac{\partial^2 B}{\partial m^2} \qquad DM2 = \delta_{11} = \frac{\partial^2 \delta}{\partial m}$$

$$BM1R1 = \beta_{12} = \frac{\partial^2 \beta}{\partial m \partial r} \qquad DM1R1 = \delta_{12} = \frac{\partial^2 \delta}{\partial m \partial r}$$

$$BR1 = \beta_2 = \frac{\partial \beta}{\partial r} \qquad DR1 = \delta_2 = \frac{\partial \delta}{\partial r}$$

$$BR2 = \beta_{22} = \frac{\partial^2 \beta}{\partial r^2} \qquad DR2 = \delta_{22} = \frac{\partial^2 \delta}{\partial r^2}$$

$$DM = \Delta m \qquad DR = \Delta r$$

$$C = C \qquad C12 = C_{12} = \frac{\partial^2 C}{\partial m \partial r}$$

$$XM = m \qquad GX = G(m, r)$$

$$R = r \qquad HX = H(m, r)$$

Because Eqs. (75) and (76) contain high power polynomial functions of m and r, an extremely large computer capacity is required. Since the maximum storage capacity of the IBM-1620 using Fortran II is limited to 60000, the program for solving the equations was separated into two subprograms (shown in Program 1).

The numerical trial and error method is illustrated as follows.

Let (m_0, r_0) be an initial approximation to a root of Eqs. (75) and (76), and let Δm and Δr be the increments of m, r respectively. Substitutions of m_0, r_0 into Eqs. (75) and (76) yields

$$G(m_0, r_0) = g(m_0, r_0) \tag{77}$$

$$H(m_0, r_0) = h(m_0, r_0) \tag{78}$$

Theoretically, both function $g(m_0, r_0)$ and function $h(m_0, r_0)$ should be equal to zero if (m_0, r_0) is an exact root of $G(m, r)$ and $H(m, r)$. This is difficult to achieve because the convergent point is rather hard to find for high degree polynomial equations. Therefore the increments Δm and Δr are added to make $G(m, r)$ and $H(m, r)$ possibly convergent. Furthermore, other initial approximations should be tried to get all possible convergent points. Care must be taken with the roots of $G(m, r)$ and $H(m, r)$, since there can be both real and complex ones. However, only real roots are of interest in this work.

Referring to the flow chart (Appendix C) for Program 1, one should supply both initial approximation and increments of m, r and set the iteration = N(1, 2, 3, . . ., n). Then one may compute polynomials and obtain computer results in subprogram A. These results are then introduced into subprogram B. Computation of these polynomials in subprogram B yields the required solutions: XM, R, GX, HX, C, C12.

There will be 11 roots because $G(m, r)$ and $H(m, r)$ are 11^{th} degree polynomial equations. The optimum process is applied to select the required roots.

APPENDIX B

Description of the IBM-1620 Computer Program Used for Program 2, Program 3, and Program 4.

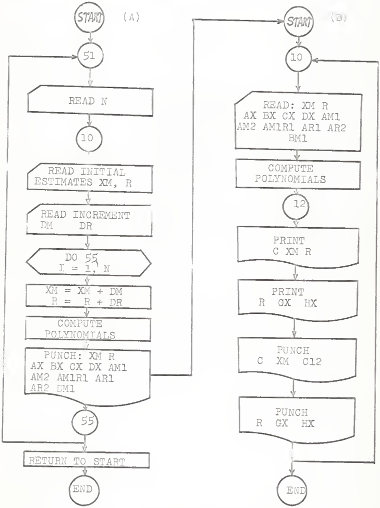
IBM-1620 FORGO is applied to calculate the evaluation of $|T(j\omega)|^2$ versus ω (angular frequency) for the equation such that

$$|T(j\omega)|^2 = \frac{a_n\omega^n + a_{n-1}\omega^{n-1} + \ldots + a_1\omega + a_0}{b_n\omega^n + b_{n-1}\omega^{n-1} + \ldots + b_1\omega + b_0} \qquad (79)$$

Referring to Program 2, Program 3, and Program 4, one can easily obtain the evaluation of $|T(j\omega)|^2$ for change of ω from zero to infinity. Computer results are shown respectively in those programs.

APPENDIX B

IBM-1620 Computer Programs Used in This Work



Flow chart for calculation of two bivariate polynomial equations in Program 1

PROGRAM 1. NUMERICAL TRIAL AND ERROR METHOD FOR CALCULATION
OF POLYNOMIAL EQUATIONS

```
C   CALCULATION OF TWO BIVARIATE POLYNOMIAL EQUATIONS
*12[illegible]
  1 FORMAT(4E15.5)
 21 FORMAT(2(E15.5))
 20 FORMAT(5(E15.5))
 52 FORMAT(I3)
 51 READ52,N
  5 FORMAT(2F9.8)
 10 READ 1,XM,R
    READ50,DM,DR
    XM=XM-DM
    R=R-DR
    DO 55I=1,N
    XM=XM+DM
    R=R+DR
 11 AX=6912 .*XM**11+219616.*XM**10+456192.*XM**10*R+274384.*XM**9+1[illegible]
   14176.*XM**9*R+1196080.*XM**9*R**2+185096.*XM**8+1454096.*XM**8*R+2
   2929920.*XM**8*R**2+1564672.*XM**8*R**3+70816.*XM**7+882888.*XM**7*
   3R+2932096.*XM**7*R**2+3246080.*XM**7*R**3+1051808.*XM**7*R**4
    AX=AX+15216.*XM**6+306880.*XM**6*R+1559888.*XM**6*R**2+2758144.*XM
   1**6*R**3+1723936.*XM**6*R**4+262144.*XM**6*R**5+1680.*XM**5+60784.
   2*XM**5*R+477320.*XM**5*R**2+1233280.*XM**5*R**3+1167360.*XM**5*R**
   34+32768 .*XM**5*R**5+72.*XM**4+6320.*XM**4*R+84432.*XM**4*R**2
    AX=AX+313922.*XM**4*R**3+409600.*XM**4*R**4+163840.*XM**4*R**5+264
   1.*XM**3*R+8056.*XM**3*R**2+46032.*XM**3*R**3+77184.*XM**3*R**4+409
   260.*XM**3*R**5+320.*XM**2*R**2+3680.*XM**2*R**3+7296.*XM**2*R**4+5
   3120.*XM**2*R**5+128.*XM*R**3+256.*XM*R**4+256.*XM*R**5
    BX=-22480.*XM**10-31544.*XM**9-138448.*XM**9*R-11780.*XM**8-210216
   1.*XM**8*R-318208.*XM**8*R**2+1358.*XM**7-84196.*XM**7*R-424928.*XM
   2**7*R**2-323584.*XM**7*R**3+1696.*XM**6-8090.*XM**6*R-207944.*XM**
   36*R**2-407808.*XM**6*R**3-12880.*XM**6*R**4+330.*XM**5
    BX=BX+3352.*XM**5*R-44088.*XM**5*R**2-206208.*XM**5*R**3-133376.*X
   1M**5*R**4+18.*XM**4+830.*XM**4*R-3206.*XM**4*R**2-59808.*XM**4*R**
   23-69888.*XM**4*R**4+1024.*XM**4*R**5+48.*XM**3*R+112.*XM**3*R**2-1
   35568.*XM**3*R**3-23872.*XM**3*R**4+6656.*XM**3*R**5+14.*XM**2*R**2
    BX=BX-1 24.*XM**2*R**3-5248.*XM**2*R**4+1536.*XM**2*R**5-48.*XM*R*
   1*3-640.*XM*R**4+128.*XM*R**5-32.*R**4
    CX=-980.*XM**9-854.*XM**8-4588.*XM**8*R-158.*XM**7-4194.*XM**7*R-7
   1112.*XM**7*R**2+33.*XM**6-928.*XM**6*R-6692.*XM**6*R**2-3648.*XM**
   26*R**3+9.*XM**5-54.*XM**5*R-2552.*XM**5*R**2-3504.*XM**5*R**3+33.*
   3XM**4*R+68.*XM**4*R**2-832.*XM**4*R**3-64.*XM**4*R**4
    CX=CX+40.*XM**3*R**2+16.*XM**3*R**3+16.*XM**2*R
    DX=-10.*XM**8-3.*XM**7-36.*XM**7*R-10.*XM**6*R-32.*XM**6*R**2-8.*X
   1M**5*R**2
    AM1=760320.*XM**10+215616 .*XM**9+4561920.*XM**9*R+2469456.*XM**8+
   111467584.*XM**8*R+1078272 .*XM**8*R**2+1480768.*XM**7+11632768.*XM
   2**7*R+2343936 .*XM**7*R**2+12517376.*XM**7*R**3+495712.*XM**6+6180
   3216.*XM**6*R+20524672.*XM**6*R**2+2272256 .*XM**6*R**3
```

```
 AM1=AM1+7[illegible]62656.*X[illegible]**6*R**4+[illegible].*X[illegible]**5+18[illegible].*XM**5*R+935[illegible].
1*XM**5*R**2+16548864.*XM**5*[illegible]+1[illegible]223616.*X[illegible]*R**4+1572864.*X[illegible]*
2*5*[illegible]**5+84[illegible].*X[illegible]**4+30392[illegible].*X[illegible] 4 [illegible] 2300000 [illegible] 4 [illegible] 2 6100400.*X
3M**4*R**3+5636[illegible].*XM**4*R**4+163[illegible].*X[illegible]**4*R**5+268.*XM**3
 AM1=AM1+25280.*XM**3*R+337728.*X[illegible]*3*R**2+125[illegible]660.*XM**3*R**3+1638
1400.*XM**3*R**4+65[illegible]360.*XM**3*R**[illegible]+7[illegible]2.*XM**2*R+24168.*XM**2*R**2+
2138[illegible]96.*XM**2*R**[illegible]+231[illegible]2.*XM**2*R**4+12288[illegible].*XM**2*R**5+640.*X[illegible]*R
3**2+736[illegible].*XM*R**3+14592.*XM*R**4+1024[illegible].*XM*R**5+128.*R**3
 AM1=AM1+256.*R**4+256.*R**5
 AM2=76[illegible]32[illegible].*XM**9+194[illegible]5440.*XM**8+41[illegible]728[illegible].*X[illegible]**8*R+19755648.*X[illegible]*
1*7+9174[illegible]672.*XM**7*R+8626176[illegible].*X[illegible]**7*R**2+1[illegible]365376.*X[illegible]**6+8142[illegible]376
2.*XM**6*R+164[illegible]7552[illegible].*XM**6*R**2+87621632.*X[illegible]**6*R**3+2974272.*XM**
35+37[illegible]81296.*XM**5*R+123148032.*XM**5*R**2+136335360.*XM**5*R**3
 AM2=AM2+44175936.*XM**5*R**4+456480.*X[illegible]**4+9206400.*XM**4*R+467966
140.*XM**4*R**[illegible]+62744320.*XM**4*R**3+51118080.*XM**4*R**4+7864320.*
2XM**4*R**5+33600.*XM**3+1215680.*XM**3*R+9546400.*XM**3*R**2+24665
36[illegible]0.*XM**3*R**3+23347200.*XM**3*R**4+65536[illegible].*XM**3*R**5
 AM2=AM2+864.*XM**2+75840.*XM**2*R+1013184.*XM**2*R**2+3767040.*XM*
1*2*R**3+4915200.*XM**2*R**4+1966080.*XM**2*R**5+1584.*XM*R+48336.*
2XM*R**2+276192.*XM*R**3+463104.*X[illegible]*R**4+24576[illegible].*XM*R**5+640.*R**2+
[illegible]736[illegible].*R**3+14592.*R**4+1024[illegible].*R**5
 AM1R1=456192[illegible].*XM**9+11467584.*XM**8+21565440.*XM**8*R+11632768.*X
M**7+4687872[illegible].*XM**7*R+37552128.*XM**7*R**2+6180216.*XM**6+4104934
24.*XM**6*R+681676[illegible].*XM**6*R**2+2945[illegible]624.*XM**6*R**3+1841280.*XM**
35+18718656.*XM**5*R+49646592.*XM**5*R**2+40094464.*XM**5*R**3
 AM1R1=AM1R1+786432[illegible].*XM**5*R**4+303920.*XM**4+4773200.*XM**4*R+184
1992[illegible].*XM**4*R**2+23347200.*XM**4*R**3+8192000.*XM**4*R**4+25280.*
2XM**3+675456.*XM**3*R+3767040.*X[illegible]**3*R**2+6553600.*XM**3*R**3+3276
3800.*XM**3*R**4+792.*XM**2+48336.*XM**2*R+414288.*XM**2*R**2
 AM1R1=AM1R1+926208.*XM**2*R**3+6144[illegible].*XM**2*R**4+1280.*XM*R+22080
1.*XM*R**2+58368.*X[illegible]*R**3+51200.*XM*R**4+384.*R**2+1024.*R**3+1280.
2*R**4
 AR1=456192.*XM**1[illegible]+1274176.*XM**9+2396160.*XM**9*R+1454096.*XM**8+
1585984[illegible].*XM**8*R+4694[illegible]16.*XM**8*R**2+882888.*XM**7+5864192.*XM**7*
2R+9738240.*XM**7*R**2+4207232.*XM**7*R**3+306880.*XM**6+3119776.*X
3M**6*R+8274432.*XM**6*R**2+6815744.*XM**6*R**3+1310720.*XM**6*R**4
 AR1=AR1+60784.*XM**5+954640.*XM**5*R+36[illegible]9840.*X[illegible]**5*R**2+4669440.*
1XM**5*R**3+163840[illegible].*XM**5*R**4+6320.*XM**4+168864.*XM**4*R+941760.
2*XM**4*R**2+163840[illegible].*X[illegible]**4*R**3+819200.*XM**4*R**4+264.*XM**3+1611
32.*XM**3*R+138096.*XM**3*R**2+308736.*XM**3*R**3
 AR1=AR1+20480[illegible].*XM**3*R**4+640.*XM**2*R+11040.*XM**2*R**2+29184.*X
1M**2*R**3+25600.*XM**2*R**4+384.*XM*R**2+1024.*XM*R**3+1280.*XM*R*
2*4
 AR2=2396160.*XM**9+5859840.*XM**8+9388[illegible]2.*XM**8*R+5864192.*XM**7+
11947648[illegible].*XM**7*R+12621696.*XM**7*R**2+3119776.*XM**6+16548864.*XM
2**6*R+2[illegible]447232.*X[illegible]**6*R**2+524288[illegible].*XM**6*R**3+954640.*XM**5+73[illegible]96
368[illegible].*XM**5*R+140[illegible]832[illegible].*X[illegible]**5*R**2+655360[illegible].*X[illegible]**5*R**3
 AR2=AR2+168864.*XM**4+188352[illegible].*XM**4*R+4915200.*XM**4*R**2+327680[illegible]
1.*XM**4*R**3+16112.*XM**3+276192.*XM**3*R+[illegible]26208.*XM**3*R**2+8192[illegible]
20.*XM**3*R**3+640.*XM**2+22[illegible]80.*XM**2*R+87552.*XM**2*R**2+102400[illegible].*
3XM**2*R**3+768.*XM*R+3072.*XM*R**2+5120.*XM*R**3
```

```
 BM1=-2246[illegible].*XM**9-[illegible].*X[illegible]-1[illegible]46[illegible]32.*XM**[illegible]-[illegible]4240.*XM**7-168
1172[illegible].*XM**7*R-2545[illegible]64.*X[illegible]**[illegible].*XM**6-[illegible]72.*XM**6*R-2974
2496.*XM**6*R**2-226[illegible]88.*X[illegible]+1[illegible]168.*XM**5-48540.*XM**5*R-1[illegible]
[illegible]47664.*XM**5*R**2-2446848.*X[illegible]**5*R**[illegible]-7[illegible]728[illegible].*XM**5*R**4
 BM1=BM1+165[illegible].*XM**4+1676[illegible].*X[illegible]**4*R-22[illegible]44[illegible].*X[illegible]**4*R**2-1031[illegible]4[illegible].*X[illegible]
1*4*R**3-6668[illegible].*X[illegible]**4*R**4+72.*X[illegible]**3+332[illegible].*X[illegible]**3*R-12824.*X[illegible]
2*2-239232.*XM**3*[illegible]**3-279552.*XM**3*R**4+4096.*X[illegible]**3*[illegible]**5+144.*X[illegible]
3*2*R+336.*XM**2*R**2-46704.*XM**2*R**3-71616.*XM**2*R**4
 BM1=BM1+19968.*XM**2*R**5+26.*XM*R**2-2[illegible]48.*XM*R**3-10496.*XM*R**4
1+3072.*XM*R**5-48.*R**3-640.*R**4-128.*R**5
 PUNCH 1,XM,R
 PUNCH 2[illegible],AX,BX,CX,DX,AM1
 PUNCH 2[illegible],AM2,AM1R1,AR1,AR2,B[illegible]1
[illegible] CONTINUE
 GO TO 51
 END
```

```
*12
 20 FORMAT(5E15.5)
  1 FORMAT(2E15.5)
  2 FORMAT(2X,2HC=F15.5,5X,3HXM=E12.5,5X,2HR=E12.5,/)
  3 FORMAT(2X,2HC=F15.5,5X,3HXM=E12.5,5X,4HC12=E12.5,/)
  4 FORMAT(2X,2HR=F15.5,6X,3HGX=E12.5,6X,3HHX=E12.5,/)
 10 READ  1,XM,R
    READ  2 ,AX,BX,CX,DX,AM1
    READ  20,AM2,AM1R1,AR1,AR2,BM1
    BM2=-2023200.*XM**8-2271168.*XM**7-9968256.*XM**7*R-659680.*XM**6-
   111772096.*XM**6*R-17819648.*XM**6*R**2+57036.*XM**5-3536232.*XM**5
   2*R-17846976.*XM**5*R**2-13590528.*XM**5*R**3+50940.*XM**4-242700.*
   3XM**4*R-6238320.*XM**4*R**2-1223424.*XM**4*R**3
    BM2=BM2-368640.*XM**4*R**4+6600.*XM**3+67040.*XM**3*R-881760.*XM*
   1*3*R**2-412416.*XM**3*R**3-266752.*XM**3*R**4+216.*XM**2+9960.*X
   2M**2*R-38472.*XM**2*R**2-717696.*XM**2*R**3-838656.*XM**2*R**4+122
   388.*XM**2*R**5+288.*XM*R+672.*XM*R**2-93408.*XM*R**3
    BM2=BM2-143232.*XM*R**4+39936.*XM*R**5+28.*R**2-2048.*R**3-10496.*
   1R**4+3072.*R**5
    BM1R1=-1246032.*XM**8-1681728.*XM**7-5991328.*XM**7*R-589372.*XM**
   16-5948992.*XM**6*R-6795264.*XM**6*R**2-48540.*XM**5-2495328.*XM**5
   2*R-7340544.*XM**5*R**2-2949120.*XM**5*R**3-16760.*XM**4-440880.*XM
   3**4*R-3093120.*XM**4*R**2-2667520.*XM**4*R**3+3320.*XM**3
    BM1R1=BM1R1-25648.*XM**3*R-717696.*XM**3*R**2-1118208.*XM**3*R**3+
   24.*XM**2*R**3+99840.*XM**2*R**4+56.*XM*R-6144.*XM*R**2-41984.*XM*R
   3**3+15360.*XM*R**4-144.*R**2-2560.*R**3+640.*R**4
    BR1=-138448.*XM**9-210216.*XM**8-636416.*XM**8*R-84196.*XM**7-8498
   156.*XM**7*R-970752.*XM**7*R**2-8090.*XM**6-415888.*XM**6*R-1223424
   2.*XM**6*R**2-491520.*XM**6*R**3+3352.*XM**5-88176.*XM**5*R-618624.
   3*XM**5*R**2-533504.*XM**5*R**3+830.*XM**4-6412.*XM**4*R
    BR1=BR1-179424.*XM**4*R**2-279552.*XM**4*R**3+5120.*XM**4*R**4+48.
   1*XM**3+224.*XM**3*R-46704.*XM**3*R**2-95488.*XM**3*R**3+33280.*XM*
   2*3*R**4+28.*XM**2*R-3072.*XM**2*R**2-20992.*XM**2*R**3+7680.*XM**2
   3*R**4-144.*XM*R**2-2560.*XM*R**3+640.*XM*R**4-128.*R**3
    BR2=-636416.*XM**8-849856.*XM**7-1941504.*XM**7*R-415888.*XM**6-24
   146848.*XM**6*R-1474560.*XM**6*R**2-88176.*XM**5-1237248.*XM**5*R-1
   260512.*XM**5*R**2-6412.*XM**4-358848.*XM**4*R-838656.*XM**4*R**2+
   32048 .*XM**4*R**3+224.*XM**3-93408.*XM**3*R-286464.*XM**3*R**2
    BR2=BR2+13312 .*XM**3*R**3+28.*XM**2-6144.*XM**2*R-62976.*XM**2*R*
   1*2+30720.*XM**2*R**3-288.*XM*R-7680.*XM*R**2+2560.*XM*R**3-384.*R*
   2*2
    CM1=-8820.*XM**8-6832.*XM**7-36704.*XM**7*R-1106.*XM**6-29358.*XM*
   1*6*R-49784.*XM**6*R**2+198.*XM**5-5568.*XM**5*R-40152.*XM**5*R**2-
   221888.*XM**5*R**3+45.*XM**4-270.*XM**4*R-12760.*XM**4*R**2-17520.*
   3XM**4*R**3+132.*XM**3*R+272.*XM**3*R**2-3328.*XM**3*R**3
    CM1=CM1-256.*XM**3*R**4+120.*XM**2*R**2+48.*XM**2*R**3+32.*XM*R**3
    CM2=-70560.*XM**7-47824.*XM**6-256928.*XM**6*R-6636.*XM**5-176148.
   1*XM**5*R-298704.*XM**5*R**2+990.*XM**4-27840.*XM**4*R-200760.*XM**
   24*R**2-109440.*XM**4*R**3+180.*XM**3-1080.*XM**3*R-51040.*XM**3*R*
   3*2-70080.*XM**3*R**3+396.*XM**2*R+816.*XM**2*R**2-9984.*XM**2*R**3
    CM2=CM2-768.*XM**2*R**4+240.*XM*R**2+96.*XM*R**3+32.*R**3
```

```
      CM1R1=-36704.*XM**7-29358.*XM**6-99568.*XM**6*R-5568.*XM**5-80304.
     1*XM**5*R-65664.*XM**5*R**2-270.*XM**4-25520.*XM**4*R-52560.*XM**4*
     2R**2+132.*XM**3+544.*XM**3*R-9984.*XM**3*R**2-1024.*XM**3*R**3+240
     3.*XM**2*R+144.*XM**2*R**2+96.*XM*R**2
      CR1=-4588.*XM**8-4194.*XM**7-14224.*XM**7*R-928.*XM**6-13384.*XM**
     16*R-10944.*XM**6*R**2-54.*XM**5-5104.*XM**5*R-10512.*XM**5*R**2+33
     22.*XM**4+136.*XM**4*R-2496.*XM**4*R**2-256.*XM**4*R**3+80.*XM**3*R+
     348.*XM**3*R**2+48.*XM**2*R**2
      CR2=-14224.*XM**7-13384.*XM**6-21888.*XM**6*R-5104.*XM**5-21024.*X
     1M**5*R+136.*XM**4-4992.*XM**4*R-768.*XM**4*R**2+80.*XM**3+96.*XM**
     23*R+96.*XM**2*R
      DM1=-80.*XM**7-21.*XM**6-252.*XM**6*R-60.*XM**5*R-192.*XM**5*R**2-
     140.*XM**4*R**2
      DM2=-560.*XM**6-126.*XM**5-1512.*XM**5*R-300.*XM**4*R-960.*XM**4*R
     1**2-160.*XM**3*R**2
      DM1R1=-252.*XM**6-60.*XM**5-384.*XM**5*R-80.*XM**4*R
      DR1=-36.*XM**7-10.*XM**6-64.*XM**6*R-16.*XM**5*R
      DR2=-64.*XM**6-16.*XM**5
      A=AX*BX*(CM2*DM1-CM1*DM2)+BM2*DX*(AX*CM1-AM1*CX)
      B=BX*CX*(AM1*DM2-AM2*DM1)+BM1*DX*(AM2*CX-AX*CM2)
      D=AX**2*(CM1*DM2-CM2*DM1)+AX*AM2*(CX*DM1-CM1*DX)+AM2*BX*CX*CM1
      E=CX**2*(AM1*BM2-AM2*BM1)+CX*CM2*(AX*BM1-AM1*BX)-AX*BM2*CX*CM1
      U=AX*BX*(CR2*DR1-CR1*DR2)+BR2*DX*(AX*CR1-AR1*CX)
      V=BX*CX*(AR1*DR2-AR2*DR1)+BR1*DX*(AR2*CX-AX*CR2)
      P=AX**2*(CR1*DR2-CR2*DR1)+AX*AR2*(CX*DR1-CR1*DX)+AR2*BX*CX*CR1
      Q=CX**2*(AR1*BR2-AR2*BR1)+CX*CR2*(AX*BR1-AR1*BX)-AX*BR2*CX*CR1
      C=(A+B)/(D+E)
      HX=AM1*C**3+BM1*C**2+CM1*C+DM1
      GX=(A+B)/(D+E)-(U+V)/(P+Q)
      WX=C**2*(AM1R1*CX-AX*CM1R1+AM1*CR1+AR1*CM1)+C*(AM1*DR1+AR1*DR1+BR1
     1*CM1+BR1*CR1+AM1R1*DX-AX*DM1R1+BM1R1*CX-BX*CM1R1)+BM1*DR1+BR1*DM1+
     2BM1R1*DX-BX*DM1R1
      WY=2.*(CX*C+DX)*(AM1*C+BM1)*(AR1*C+BR1)/(AX*C+BX)
      WZ=2.*C*(AX*C+BX)**2-AX*DX+BX*CX
      C12=(WX-WY)/WZ
   12 PRINT 2,C,XM,R
      PRINT 4,R,GX,HX
   13 PUNCH 3,C,XM,C12
      PUNCH 4,R,GX,HX
      GO TO 10
      END
```

INPUT DATA OF PROGRAM 1.

```
  5
     .00200E-00    0.50200E-00
+.001    +.001

  5
     .01200E-00    0.50000E-00
+.01     +.01

  5
     .00100E-00    0.50200E-00
- .0001  -0.0001

  5
     .35400E-00    0.04000E-00
+.001    +.001

  5
     .37000E-00    0.02100E-00
 .005    0.01

  5
     .32450E-00    0.17900E-00
- .0001  +0.0

  5
     .32450E-00    0.17900E-00
+ .0     +0.01

  1
     .50200E-00    0.00100E-00
+ .0     +0.0

  1
     .34390E-00    0.03790E-00
+ .0     +0.0

  3
     .35450E-00    0.04250E-00
- .005   +0.001

  3
     .33450E-00    0.02790E-00
- .005   -0.005

  3
     .32420E-00    0.01790E-00
+ .01    +0.
```

COMPUTER RESULTS OF PROGRAM 1.

C= 5614.91378E+11	XM= 2.00000E-03	C12=-3.87070E-06
R= 5020.00000E-04	GX= 7.88194E-09	HX= 2.98650E-10
C= 2001.26781E-10	XM= 3.00000E-03	C12=-2.06148E-05
R= 5030.00000E-04	GX= 2.98780E-08	HX= 1.66781E-09
C= 4998.09563E-10	XM= 4.00000E-03	C12=-6.02374E-05
R= 5040.00000E-04	GX= 7.72587E-08	HX= 5.77638E-09
C= 1026.14803E-09	XM= 5.00000E-03	C12=-1.34925E-04
R= 5050.00000E-04	GX= 1.60096E-07	HX= 1.53600E-08
C= 1859.60756E-09	XM= 6.00000E-03	C12=-2.58965E-04
R= 5060.00000E-04	GX= 2.85099E-07	HX= 3.44926E-08
C= 1946.70853E-08	XM= 1.20000E-02	C12=-3.34778E-03
R= 5000.00000E-04	GX= 8.25184E-06	HX= 7.72637E-07
C= 1462.97428E-07	XM= 2.20000E-02	C12=-3.19896E-02
R= 5100.00000E-04	GX=-3.42133E-05	HX= 1.10602E-05
C= 4810.84067E-07	XM= 3.20000E-02	C12=-1.58675E-01
R= 5200.00000E-04	GX= 8.93279E-04	HX= 4.20783E-05
C= 1112.07029E-06	XM= 4.20000E-02	C12=-1.25363E-00
R= 5300.00000E-04	GX= 1.38002E-03	HX= 3.95923E-05
C= 2173.34047E-06	XM= 5.20000E-02	C12= 9.15740E-01
R= 5400.00000E-04	GX= 2.43379E-03	HX=-3.32558E-04
C= 6601.89946E-12	XM= 1.00000E-03	C12= 2.03680E-07
R= 5020.00000E-04	GX= 6.41334E-10	HX= 1.68023E-11
C= 4784.87552E+12	XM= 9.00000E-04	C12= 2.53433E-07
R= 5019.00000E-04	GX= 4.58203E-10	HX= 1.09037E-11

C= 1054.58622E-05	XM= 3.54000E-01	C12=-1.02713E+01
R= 4000.00000E-05	GX= 4.49330E-02	HX=-2.80293E-01
C= 1062.72215E-05	XM= 3.55000E-01	C12=-9.23096E-00
R= 4100.00000E-05	GX= 4.46904E-02	HX=-2.89962E-01
C= 1070.46877E-05	XM= 3.56000E-01	C12=-8.32577E-00
R= 4200.00000E-05	GX= 4.44540E-02	HX=-2.99867E-01
C= 1077.83685E-05	XM= 3.57000E-01	C12=-7.53435E-00
R= 4300.00000E-05	GX= 4.42231E-02	HX=-3.10012E-01
C= 1084.83631E-05	XM= 3.58000E-01	C12=-6.83936E-00
R= 4400.00000E-05	GX= 4.39971E-02	HX=-3.20401E-01
C= 6045.59426E-06	XM= 3.70000E-01	C12=-1.50812E+01
R= 2100.00000E-05	GX= 4.49355E-02	HX=-2.40059E-01
C= 7566.30695E-06	XM= 3.75000E-01	C12=-6.99655E-00
R= 3100.00000E-05	GX= 4.12903E-02	HX=-3.18245E-01
C= 8743.54689E-06	XM= 3.80000E-01	C12=-3.72768E-00
R= 4100.00000E-05	GX= 3.96779E-02	HX=-4.11281E-01
C= 9642.64908E-06	XM= 3.85000E-01	C12=-2.19897E-00
R= 5100.00000E-05	GX= 3.86665E-02	HX=-5.20351E-01
C= 1030.70943E-05	XM= 3.90000E-01	C12=-1.42012E-00
R= 6100.00000E-05	GX= 3.78118E-02	HX=-6.46227E-01
C= 1054.58622E-05	XM= 3.54000E-01	C12=-1.02713E+01
R= 4000.00000E-05	GX= 4.49330E-02	HX=-2.80293E-01
C= 1053.75083E-05	XM= 3.53900E-01	C12=-1.03838E+01
R= 3990.00000E-05	GX= 4.49577E-02	HX=-2.79339E-01
C= 1075.54104E-05	XM= 3.48900E-01	C12=-1.42297E+01
R= 3890.00000E-05	GX= 4.62912E-02	HX=-2.53947E-01

C= 7864.13244E-06	XM= 3.24500E-01	C12= 9.33546E+01
R= 1790.00000E-05	GX= 5.84102E-02	HX=-1.05281E-01
C= 8157.52417E-06	XM= 3.24500E-01	C12= 9.58772E+01
R= 1890.00000E-05	GX= 5.78839E-02	HX=-1.08110E-01
C= 8441.94571E-06	XM= 3.24500E-01	C12= 9.89806E+01
R= 1990.00000E-05	GX= 5.74477E-02	HX=-1.10957E-01
C= 8717.59689E-06	XM= 3.24500E-01	C12= 1.02788E+02
R= 2090.00000E-05	GX= 5.70875E-02	HX=-1.13822E-01
C= 8984.67593E-06	XM= 3.24500E-01	C12= 1.07471E+02
R= 2190.00000E-05	GX= 5.67914E-02	HX=-1.16706E-01
C= 2044.28003E-06	XM= 3.24500E-00	C12=-3.81504E-03
R= 1790.00000E-05	GX= 1.25790E-02	HX=-6.70091E+05
C= 2044.29912E-06	XM= 3.24400E-00	C12=-3.81791E-03
R= 1790.00000E-05	GX= 1.25817E-02	HX=-6.68566E+05
C= 2044.31909E-06	XM= 3.24300E-00	C12=-3.82078E-03
R= 1790.00000E-05	GX= 1.25845E-02	HX=-6.67045E+05
C= 2044.33876E-06	XM= 3.24200E-00	C12=-3.82366E-03
R= 1790.00000E-05	GX= 1.25873E-02	HX=-6.65526E+05
C= 2044.35791E-06	XM= 3.24100E-00	C12=-3.82653E-03
R= 1790.00000E-05	GX= 1.25901E-02	HX=-6.64011E+05
C= 1977.08684E-06	XM= 5.02000E-01	C12=-1.29676E-00
R= 1000.00000E-06	GX=-9.60148E-02	HX=-1.18903E-00

C= 1096.18200E-05	XM= 3.43900E-01	C12=-2.04427E+01
R= 3790.00000E-05	GX= 4.77748E-02	HX=-2.30236E-01
C= 1089.69948E-05	XM= 3.54500E-01	C12=-8.62141E-00
R= 4250.00000E-05	GX= 4.48423E-02	HX=-2.96020E-01
C= 1142.95223E-05	XM= 3.49500E-01	C12=-1.01324E+01
R= 4350.00000E-05	GX= 4.62297E-02	HX=-2.79586E-01
C= 1194.26414E-05	XM= 3.44500E-01	C12=-1.20154E+01
R= 4450.00000E-05	GX= 4.77956E-02	HX=-2.63286E-01
C= 1091.75240E-05	XM= 3.44500E-01	C12=-1.96992E+01
R= 3790.00000E-05	GX= 4.75856E-02	HX=-2.32447E-01
C= 1038.67725E-05	XM= 3.39500E-01	C12=-4.54421E+01
R= 3290.00000E-05	GX= 4.92179E-02	HX=-1.93753E-01
C= 9726.00035E-06	XM= 3.34500E-01	C12=-2.29778E+02
R= 2790.00000E-05	GX= 5.12844E-02	HX=-1.59976E-01
C= 8902.15514E-06	XM= 3.29500E-01	C12= 1.75740E+02
R= 2290.00000E-05	GX= 5.41146E-02	HX=-1.30648E-01
C= 7864.13244E-06	XM= 3.24500E-01	C12= 9.33546E+01
R= 1790.00000E-05	GX= 5.84102E-02	HX=-1.05281E-01
C= 7883.41285E-06	XM= 3.24200E-01	C12= 9.24051E+01
R= 1790.00000E-05	GX= 5.85658E-02	HX=-1.04715E-01
C= 8535.95669E-06	XM= 3.14200E-01	C12= 7.45334E+01
R= 1790.00000E-05	GX= 6.51697E-02	HX=-8.70843E-02
C= 9185.78979E-06	XM= 3.04200E-01	C12= 6.74310E+01
R= 1790.00000E-05	GX= 7.63863E-02	HX=-7.16546E-02

PROGRAM 2. CALCULATION OF THE SPECTRUM OF AN EQUAL AMPLITUDES NOTCH FILTER OF THE GOLDMAN TYPE

```
C   C  EQUAL AMPLITUDES NOTCH FILTER OF THE GOLDMAN TYPE
     1 READ, W1, W2, DELW
       PUNCH 4
     4 FORMAT(11X, 5HOMEGA10X,4HEVAL/)
     3 W = W1
       X=(1.0-0.41421*W**2)**2+(0.292895*W-0.121315*W**3)**2
       Y=(1.0-1.82842*W**2)**2+(3.707105*W-0.121315*W**3)**2
       EVAL=X/Y
       PUNCH 5 , W  , EVAL
     5 FORMAT ( 5X, F12.6 , 5X, F12.6)
       W1 = W1 + DELW
       IF ( W2 - W ) 2,3,3
     2 GO TO 1
       END
```

C C DATA FOR GRAPH OF VALUE VS OMEGA - EQUAL AMPLITUDES
NOTCH FILTER OF THE GOLDMAN TYPE

OMEGA	EVAL
.000000	1.000000
.050000	.973581
.100000	.901446
.150000	.800658
.200000	.689567
.250000	.581728
.300000	.484553
.350000	.400739
.400000	.330231
.450000	.271696
.500000	.223398
.550000	.183617
.600000	.150833
.650000	.123766
.700000	.101366
.750000	.082787
.800000	.067348
.850000	.054500
.900000	.043804
.950000	.034903
1.000000	.027509
1.050000	.021386
1.100000	.016340
1.150000	.012211
1.200000	.008867
1.250000	.006196
1.300000	.004106
1.350000	.002518
1.400000	.001366
1.450000	.000594
1.500000	.000152
1.550000	.000000
1.600000	.000103
1.650000	.000429
1.700000	.000952
1.750000	.001648
1.800000	.002499
1.850000	.003485
1.900000	.004593
1.950000	.005809
2.000000	.007120
2.050000	.008517
2.100000	.009992
2.150000	.011535
2.200000	.013140
2.250000	.014801
2.300000	.016513
2.350000	.018270
2.400000	.020069
2.450000	.021905

OMEGA	EVAL
2.500000	.023777
2.550000	.025679
2.600000	.027611
2.650000	.029570
2.700000	.031554
2.750000	.033560
2.800000	.035588
2.850000	.037637
2.900000	.039704
2.950000	.041788
3.000000	.043890
3.050000	.046007
3.100000	.048139
3.150000	.050286
3.200000	.052447
3.250000	.054621
3.300000	.056808
3.350000	.059007
3.400000	.061217
3.450000	.063440
3.500000	.065674
3.550000	.067918
3.600000	.070173
3.650000	.072439
3.700000	.074715
3.750000	.077000
3.800000	.079296
3.850000	.081601
3.900000	.083915
3.950000	.086239
4.000000	.088572
4.050000	.090913
4.100000	.093263
4.150000	.095622
4.200000	.097989
4.250000	.100365
4.300000	.102748
4.350000	.105140
4.400000	.107539
4.450000	.109946
4.500000	.112361
4.550000	.114783
4.600000	.117213
4.650000	.119649
4.700000	.122093
4.750000	.124543
4.800000	.127000
4.850000	.129464
4.900000	.131934
4.950000	.134410
5.000000	.136893

OMEGA	EVAL
.000000	1.000000
1.000000	.027509
2.000000	.007120
3.000000	.043890
4.000000	.088572
5.000000	.136893
6.000000	.187582
7.000000	.239365
8.000000	.290978
9.000000	.341341
10.000000	.389635
11.000000	.435303
12.000000	.478020
13.000000	.517647
14.000000	.554178
15.000000	.587703
16.000000	.618370
17.000000	.646365
18.000000	.671889
19.000000	.695146
20.000000	.716338
21.000000	.735654
22.000000	.753274
23.000000	.769362
24.000000	.784066
25.000000	.797525
26.000000	.809859
27.000000	.821179
28.000000	.831584
29.000000	.841162
30.000000	.849992
31.000000	.858146
32.000000	.865686
33.000000	.872668
34.000000	.879145
35.000000	.885160
36.000000	.890755
37.000000	.895966
38.000000	.900826
39.000000	.905366
40.000000	.909610
41.000000	.913584
42.000000	.917310
43.000000	.920806
44.000000	.924091
45.000000	.927181
46.000000	.930090
47.000000	.932833
48.000000	.935422
49.000000	.937866
50.000000	.940178

OMEGA	EVAL
50.000000	.940178
60.000000	.957672
70.000000	.968544
80.000000	.975735
90.000000	.980728
100.000000	.984332
110.000000	.987015
120.000000	.989066
130.000000	.990668
140.000000	.991943
150.000000	.992974
160.000000	.993820
170.000000	.994522
180.000000	.995110
190.000000	.995609
200.000000	.996036
210.000000	.996403
220.000000	.996721
230.000000	.996999
240.000000	.997244
250.000000	.997459
260.000000	.997650
270.000000	.997821
280.000000	.997973
290.000000	.998110
300.000000	.998234
310.000000	.998346
320.000000	.998448
330.000000	.998540
340.000000	.998625
350.000000	.998702
360.000000	.998773
370.000000	.998838
380.000000	.998899
390.000000	.998954
400.000000	.999006
410.000000	.999054
420.000000	.999098
430.000000	.999140
440.000000	.999179
450.000000	.999215
460.000000	.999248
470.000000	.999280
480.000000	.999310
490.000000	.999337
500.000000	.999364

OMEGA	EVAL
500.000000	.999364
600.000000	.999558
700.000000	.999675
800.000000	.999751
900.000000	.999803
1000.000000	.999841
1100.000000	.999868
1200.000000	.999889
1300.000000	.999906
1400.000000	.999919
1500.000000	.999929
1600.000000	.999938
1700.000000	.999945
1800.000000	.999951
1900.000000	.999956
2000.000000	.999960
2100.000000	.999964
2200.000000	.999967
2300.000000	.999970
2400.000000	.999972
2500.000000	.999975
2600.000000	.999976
2700.000000	.999978
2800.000000	.999980
2900.000000	.999981
3000.000000	.999982
3100.000000	.999983
3200.000000	.999984
3300.000000	.999985
3400.000000	.999986
3500.000000	.999987
3600.000000	.999988
3700.000000	.999988
3800.000000	.999989
3900.000000	.999990
4000.000000	.999990
4100.000000	.999991
4200.000000	.999991
4300.000000	.999991
4400.000000	.999992
4500.000000	.999992
4600.000000	.999993
4700.000000	.999993
4800.000000	.999993
4900.000000	.999993
5000.000000	.999994
5100.000000	.999994
5200.000000	.999994
5300.000000	.999994
5400.000000	.999995

OMEGA	EVAL
5500.000000	.999995
5600.000000	.999995
5700.000000	.999995
5800.000000	.999995
5900.000000	.999995
6000.000000	.999996
6100.000000	.999996
6200.000000	.999996
6300.000000	.999996
6400.000000	.999996
6500.000000	.999996
6600.000000	.999996
6700.000000	.999996
6800.000000	.999997
6900.000000	.999997
7000.000000	.999997
7100.000000	.999997
7200.000000	.999997
7300.000000	.999997
7400.000000	.999997
7500.000000	.999997
7600.000000	.999997
7700.000000	.999997
7800.000000	.999997
7900.000000	.999997
8000.000000	.999998
8100.000000	.999998
8200.000000	.999998
8300.000000	.999998
8400.000000	.999998
8500.000000	.999998
8600.000000	.999998
8700.000000	.999998
8800.000000	.999998
8900.000000	.999998
9000.000000	.999998
9100.000000	.999998
9200.000000	.999998
9300.000000	.999998
9400.000000	.999998
9500.000000	.999998
9600.000000	.999998
9700.000000	.999998
9800.000000	.999998
9900.000000	.999998
10000.000000	.999998
10100.000000	.999998
10200.000000	.999998
10300.000000	.999998
10400.000000	.999999
10500.000000	.999999

PROGRAM 3. CALCULATION OF THE SPECTRUM OF AN UNEQUAL AMPLITUDES NOTCH FILTER

```
C   C  UNEQUAL AMPLITUDES NOTCH FILTER
     1 READ, W1, W2, DELW
       PUNCH 4
     4 FORMAT(11X, 5HOMEGA10X,4HEVAL/)
     3 W = W1
       X =144.*W**6+2766.25*W**4+5172.*W**2+256.
       EVAL=(2.25*W**6-2.*W**4-92.*W**2+256.)/X
       PUNCH 5 , W  , EVAL
     5 FORMAT ( 5X, F12.6 , 5X, F12.6)
       W1 = W1 + DELW
       IF ( W2 - W ) 2,3,3
     2 GO TO 1
       END
```

C C DATA FOR GRAPH OF VALUE VS OMEGA - UNEQUAL AMPLITUDES NOTCH FILTER

OMEGA	EVAL
.000000	1.000000
.050000	.951004
.100000	.828190
.150000	.679369
.200000	.539929
.250000	.424075
.300000	.332941
.350000	.262743
.400000	.208878
.450000	.167358
.500000	.135088
.550000	.109763
.600000	.089695
.650000	.073644
.700000	.060697
.750000	.050175
.800000	.041565
.850000	.034480
.900000	.028621
.950000	.023754
1.000000	.019698
1.050000	.016309
1.100000	.013470
1.150000	.011089
1.200000	.009091
1.250000	.007414
1.300000	.006006
1.350000	.004828
1.400000	.003843
1.450000	.003022
1.500000	.002342
1.550000	.001783
1.600000	.001326
1.650000	.000957
1.700000	.000664
1.750000	.000436
1.800000	.000264
1.850000	.000141
1.900000	.000059
1.950000	.000014
2.000000	0.000000
2.050000	.000013
2.100000	.000049
2.150000	.000105
2.200000	.000178
2.250000	.000267
2.300000	.000368
2.350000	.000479
2.400000	.000601
2.450000	.000730

OMEGA	EVAL
2.500000	.000866
2.550000	.001007
2.600000	.001153
2.650000	.001304
2.700000	.001457
2.750000	.001612
2.800000	.001770
2.850000	.001929
2.900000	.002089
2.950000	.002250
3.000000	.002411
3.050000	.002572
3.100000	.002733
3.150000	.002893
3.200000	.003053
3.250000	.003212
3.300000	.003370
3.350000	.003526
3.400000	.003682
3.450000	.003836
3.500000	.003988
3.550000	.004139
3.600000	.004289
3.650000	.004437
3.700000	.004583
3.750000	.004727
3.800000	.004870
3.850000	.005010
3.900000	.005150
3.950000	.005287
4.000000	.005422
4.050000	.005556
4.100000	.005688
4.150000	.005817
4.200000	.005945
4.250000	.006072
4.300000	.006196
4.350000	.006319
4.400000	.006440
4.450000	.006559
4.500000	.006676
4.550000	.006791
4.600000	.006905
4.650000	.007017
4.700000	.007128
4.750000	.007236
4.800000	.007343
4.850000	.007449
4.900000	.007553
4.950000	.007655
5.000000	.007755

OMEGA	EVAL
.000000	1.000000
1.000000	.019698
2.000000	0.000000
3.000000	.002411
4.000000	.005422
5.000000	.007755
6.000000	.009469
7.000000	.010725
8.000000	.011657
9.000000	.012360
10.000000	.012900
11.000000	.013320
12.000000	.013653
13.000000	.013921
14.000000	.014139
15.000000	.014318
16.000000	.014468
17.000000	.014593
18.000000	.014700
19.000000	.014791
20.000000	.014869
21.000000	.014937
22.000000	.014996
23.000000	.015048
24.000000	.015094
25.000000	.015135
26.000000	.015171
27.000000	.015203
28.000000	.015232
29.000000	.015258
30.000000	.015282
31.000000	.015303
32.000000	.015323
33.000000	.015341
34.000000	.015357
35.000000	.015372
36.000000	.015386
37.000000	.015398
38.000000	.015410
39.000000	.015421
40.000000	.015431
41.000000	.015440
42.000000	.015449
43.000000	.015457
44.000000	.015464
45.000000	.015471
46.000000	.015478
47.000000	.015484
48.000000	.015490
49.000000	.015495
50.000000	.015500

OMEGA	EVAL
50.000000	.015500
60.000000	.015538
70.000000	.015561
80.000000	.015576
90.000000	.015586
100.000000	.015594
110.000000	.015599
120.000000	.015603
130.000000	.015606
140.000000	.015609
150.000000	.015611
160.000000	.015613
170.000000	.015614
180.000000	.015615
190.000000	.015616
200.000000	.015617
210.000000	.015618
220.000000	.015619
230.000000	.015619
240.000000	.015620
250.000000	.015620
260.000000	.015620
270.000000	.015621
280.000000	.015621
290.000000	.015621
300.000000	.015622
310.000000	.015622
320.000000	.015622
330.000000	.015622
340.000000	.015622
350.000000	.015622
360.000000	.015623
370.000000	.015623
380.000000	.015623
390.000000	.015623
400.000000	.015623
410.000000	.015623
420.000000	.015623
430.000000	.015623
440.000000	.015623
450.000000	.015623
460.000000	.015624
470.000000	.015624
480.000000	.015624
490.000000	.015624
500.000000	.015624
510.000000	.015624

OMEGA	EVAL
500.000000	.015624
600.000000	.015624
700.000000	.015624
800.000000	.015625
900.000000	.015625
1000.000000	.015625
1100.000000	.015625
1200.000000	.015625
1300.000000	.015625
1400.000000	.015625
1500.000000	.015625
1600.000000	.015625
1700.000000	.015625
1800.000000	.015625
1900.000000	.015625
2000.000000	.015625
2100.000000	.015625
2200.000000	.015625
2300.000000	.015625
2400.000000	.015625
2500.000000	.015625
2600.000000	.015625
2700.000000	.015625
2800.000000	.015625
2900.000000	.015625
3000.000000	.015625
3100.000000	.015625
3200.000000	.015625
3300.000000	.015625
3400.000000	.015625
3500.000000	.015625
3600.000000	.015625
3700.000000	.015625
3800.000000	.015625
3900.000000	.015625
4000.000000	.015625
4100.000000	.015625
4200.000000	.015625
4300.000000	.015625
4400.000000	.015625
4500.000000	.015625
4600.000000	.015625
4700.000000	.015625
4800.000000	.015625
4900.000000	.015625
5000.000000	.015625
5100.000000	.015625
5200.000000	.015625
5300.000000	.015625
5400.000000	.015625

PROGRAM 4. CALCULATION OF THE SPECTRUM OF AN EQUAL AMPLITUDES NOTCH FILTER

```
C  C  EQUAL AMPLITUDES NOTCH FILTER
    1 READ9,W1, W2, DELW, A
    9 FORMAT(3E10.4,F2.0)
      IF ( A ) 3,3,6
    3 W = W1
    4 FORMAT(11X, 5HOMEGA10X,4HEVAL/)
      Y = 0.5999928*W**4-28.7633238*W**2+322.3679696
      Z = 0.03053742*W**5-3.8378482*W**3+165.084187*W
      X = 7.4503121*W**4-677.1501976*W**2+322.3679696
      P = 0.03053742*W**5-122.384064*W**3+1291.817275*W
      EVAL = (Y**2+Z**2)/(X**2+P**2)
      PUNCH 5 , W  , EVAL
    5 FORMAT ( 5X, F12.6 , 5X, E14.6 )
      W1 = W1 + DELW
      IF ( W2 - C ) 2,3,3
    2 GO TO 1
    6 STOP
      END
```

C C DATA FOR GRAPH OF VALUE VS OMEGA - EQUAL AMPLILUDES NOTCH FILTER

OMEGA	EVAL
.000000	0.100000E+01
.050000	0.971406
.100000	0.894633
.150000	0.790440
.200000	0.679522
.250000	0.575533
.300000	0.484708
.350000	0.408377
.400000	0.345445
.450000	0.293955
.500000	0.251858
.550000	0.217331
.600000	0.188859
.650000	0.165226
.700000	0.145470
.750000	0.128837
.800000	0.114735
.850000	0.102696
.900000	0.923526E-01
.950000	0.834105E-01
1.000000	0.756351E-01
1.050000	0.688373E-01
1.100000	0.628638E-01
1.150000	0.575894E-01
1.200000	0.529111E-01
1.250000	0.487442E-01
1.300000	0.450179E-01
1.350000	0.416732E-01
1.400000	0.386606E-01
1.450000	0.359380E-01
1.500000	0.334699E-01
1.550000	0.312259E-01
1.600000	0.291800E-01
1.650000	0.273098E-01
1.700000	0.255960E-01
1.750000	0.240219E-01
1.800000	0.225727E-01
1.850000	0.212359E-01
1.900000	0.200001E-01
1.950000	0.188557E-01
2.000000	0.177938E-01
2.050000	0.168069E-01
2.100000	0.158881E-01
2.150000	0.150315E-01
2.200000	0.142316E-01
2.250000	0.134836E-01
2.300000	0.127832E-01
2.350000	0.121265E-01
2.400000	0.115100E-01
2.450000	0.109307E-01

OMEGA	EVAL
2.500000	0.103855E-01
2.550000	0.987206E-02
2.600000	0.938794E-02
2.650000	0.893103E-02
2.700000	0.849941E-02
2.750000	0.809131E-02
2.800000	0.770513E-02
2.850000	0.733939E-02
2.900000	0.699275E-02
2.950000	0.666398E-02
3.000000	0.635192E-02
3.050000	0.605555E-02
3.100000	0.577389E-02
3.150000	0.550605E-02
3.200000	0.525122E-02
3.250000	0.500863E-02
3.300000	0.477758E-02
3.350000	0.455742E-02
3.400000	0.434754E-02
3.450000	0.414737E-02
3.500000	0.395640E-02
3.550000	0.377413E-02
3.600000	0.360010E-02
3.650000	0.343391E-02
3.700000	0.327514E-02
3.750000	0.312343E-02
3.800000	0.297842E-02
3.850000	0.283980E-02
3.900000	0.270725E-02
3.950000	0.258049E-02
4.000000	0.245926E-02
4.050000	0.234329E-02
4.100000	0.223234E-02
4.150000	0.212621E-02
4.200000	0.202466E-02
4.250000	0.192750E-02
4.300000	0.183455E-02
4.350000	0.174562E-02
4.400000	0.166054E-02
4.450000	0.157916E-02
4.500000	0.150133E-02
4.550000	0.142689E-02
4.600000	0.135572E-02
4.650000	0.128769E-02
4.700000	0.122266E-02
4.750000	0.116054E-02
4.800000	0.110120E-02
4.850000	0.104454E-02
4.900000	0.990459E-03
4.950000	0.938864E-03
5.000000	0.889661E-03

OMEGA	EVAL
5.050000	0.842765E-03
5.100000	0.798091E-03
5.150000	0.755561E-03
5.200000	0.715099E-03
5.250000	0.676633E-03
5.300000	0.640094E-03
5.350000	0.605414E-03
5.400000	0.572531E-03
5.450000	0.541385E-03
5.500000	0.511916E-03
5.550000	0.484070E-03
5.600000	0.457792E-03
5.650000	0.433031E-03
5.700000	0.409739E-03
5.750000	0.387867E-03
5.800000	0.367370E-03
5.850000	0.348206E-03
5.900000	0.330331E-03
5.950000	0.313705E-03
6.000000	0.298290E-03
6.050000	0.284048E-03
6.100000	0.270943E-03
6.150000	0.258941E-03
6.200000	0.248008E-03
6.250000	0.238111E-03
6.300000	0.229221E-03
6.350000	0.221307E-03
6.400000	0.214341E-03
6.450000	0.208294E-03
6.500000	0.203139E-03
6.550000	0.198852E-03
6.600000	0.195407E-03
6.650000	0.192779E-03
6.700000	0.190946E-03
6.750000	0.189885E-03
6.800000	0.189574E-03
6.850000	0.189992E-03
6.900000	0.191120E-03
6.950000	0.192936E-03
7.000000	0.195423E-03
7.050000	0.198562E-03
7.100000	0.202336E-03
7.150000	0.206726E-03
7.200000	0.211717E-03
7.250000	0.217292E-03
7.300000	0.223436E-03
7.350000	0.230133E-03
7.400000	0.237370E-03
7.450000	0.245131E-03
7.500000	0.253404E-03
7.550000	0.262175E-03
7.600000	0.271431E-03

OMEGA	EVAL
7.650000	0.281160E-03
7.700000	0.291349E-03
7.750000	0.301988E-03
7.800000	0.313064E-03
7.850000	0.324567E-03
7.900000	0.336487E-03
7.950000	0.348812E-03
8.000000	0.361533E-03
8.050000	0.374639E-03
8.100000	0.388123E-03
8.150000	0.401975E-03
8.200000	0.416185E-03
8.250000	0.430745E-03
8.300000	0.445648E-03
8.350000	0.460884E-03
8.400000	0.476446E-03
8.450000	0.492327E-03
8.500000	0.508519E-03
8.550000	0.525015E-03
8.600000	0.541808E-03
8.650000	0.558892E-03
8.700000	0.576259E-03
8.750000	0.593904E-03
8.800000	0.611820E-03
8.850000	0.630002E-03
8.900000	0.648443E-03
8.950000	0.667138E-03
9.000000	0.686082E-03
9.050000	0.705270E-03
9.100000	0.724695E-03
9.150000	0.744354E-03
9.200000	0.764241E-03
9.250000	0.784352E-03
9.300000	0.804681E-03
9.350000	0.825226E-03
9.400000	0.845981E-03
9.450000	0.866942E-03
9.500000	0.888106E-03
9.550000	0.909468E-03
9.600000	0.931024E-03
9.650000	0.952771E-03
9.700000	0.974706E-03
9.750000	0.996824E-03
9.800000	0.101912E-02
9.850000	0.104160E-02
9.900000	0.106425E-02
9.950000	0.108707E-02
10.000000	0.111006E-02
10.050000	0.113321E-02

OMEGA	EVAL
10.000000	0.111006E-02
11.000000	0.160113E-02
12.000000	0.214152E-02
13.000000	0.272152E-02
14.000000	0.333604E-02
15.000000	0.398252E-02
16.000000	0.465989E-02
17.000000	0.536785E-02
18.000000	0.610650E-02
19.000000	0.687617E-02
20.000000	0.767725E-02
21.000000	0.851013E-02
22.000000	0.937520E-02
23.000000	0.102728E-01
24.000000	0.112031E-01
25.000000	0.121664E-01
26.000000	0.131629E-01
27.000000	0.141926E-01
28.000000	0.152556E-01
29.000000	0.163520E-01
30.000000	0.174816E-01
31.000000	0.186446E-01
32.000000	0.198408E-01
33.000000	0.210700E-01
34.000000	0.223322E-01
35.000000	0.236272E-01
36.000000	0.249549E-01
37.000000	0.263150E-01
38.000000	0.277073E-01
39.000000	0.291316E-01
40.000000	0.305878E-01
41.000000	0.320754E-01
42.000000	0.335943E-01
43.000000	0.351442E-01
44.000000	0.367248E-01
45.000000	0.383359E-01
46.000000	0.399771E-01
47.000000	0.416481E-01
48.000000	0.433486E-01
49.000000	0.450782E-01
50.000000	0.468368E-01
51.000000	0.486239E-01
52.000000	0.504391E-01
53.000000	0.522822E-01
54.000000	0.541528E-01
55.000000	0.560505E-01
56.000000	0.579750E-01
57.000000	0.599259E-01
58.000000	0.619029E-01
59.000000	0.639055E-01
60.000000	0.659335E-01

OMEGA	EVAL
61.000000	0.679864E-01
62.000000	0.700638E-01
63.000000	0.721654E-01
64.000000	0.742908E-01
65.000000	0.764396E-01
66.000000	0.786114E-01
67.000000	0.808059E-01
68.000000	0.830226E-01
69.000000	0.852610E-01
70.000000	0.875210E-01
71.000000	0.898020E-01
72.000000	0.921037E-01
73.000000	0.944256E-01
74.000000	0.967674E-01
75.000000	0.991286E-01
76.000000	0.101509
77.000000	0.103908
78.000000	0.106325
79.000000	0.108760
80.000000	0.111213
81.000000	0.113683
82.000000	0.116169
83.000000	0.118672
84.000000	0.121190
85.000000	0.123725
86.000000	0.126274
87.000000	0.128838
88.000000	0.131416
89.000000	0.134008
90.000000	0.136614
91.000000	0.139233
92.000000	0.141865
93.000000	0.144509
94.000000	0.147166
95.000000	0.149834
96.000000	0.152513
97.000000	0.155203
98.000000	0.157904
99.000000	0.160615
100.000000	0.163336
101.000000	0.166066
102.000000	0.168806
103.000000	0.171554
104.000000	0.174310
105.000000	0.177075
106.000000	0.179848
107.000000	0.182627
108.000000	0.185414
109.000000	0.188208
110.000000	0.191008
111.000000	0.193814
112.000000	0.196625

OMEGA	EVAL
113.000000	0.199443
114.000000	0.202265
115.000000	0.205092
116.000000	0.207923
117.000000	0.210759
118.000000	0.213598
119.000000	0.216441
120.000000	0.219287
121.000000	0.222136
122.000000	0.224988
123.000000	0.227842
124.000000	0.230698
125.000000	0.233557
126.000000	0.236416
127.000000	0.239277
128.000000	0.242139
129.000000	0.245001
130.000000	0.247865
131.000000	0.250728
132.000000	0.253591
133.000000	0.256454
134.000000	0.259317
135.000000	0.262178
136.000000	0.265039
137.000000	0.267898
138.000000	0.270756
139.000000	0.273613
140.000000	0.276467
141.000000	0.279319
142.000000	0.282169
143.000000	0.285017
144.000000	0.287861
145.000000	0.290703
146.000000	0.293542
147.000000	0.296377
148.000000	0.299208
149.000000	0.302036
150.000000	0.304860
151.000000	0.307680
152.000000	0.310496
153.000000	0.313307
154.000000	0.316113
155.000000	0.318915
156.000000	0.321712
157.000000	0.324503
158.000000	0.327290
159.000000	0.330071
160.000000	0.332846
161.000000	0.335616
162.000000	0.338379
163.000000	0.341137
164.000000	0.343889

OMEGA	EVAL
165.000000	0.346634
166.000000	0.349373
167.000000	0.352106
168.000000	0.354832
169.000000	0.357551
170.000000	0.360263
171.000000	0.362968
172.000000	0.365666
173.000000	0.368357
174.000000	0.371040
175.000000	0.373716
176.000000	0.376385
177.000000	0.379046
178.000000	0.381699
179.000000	0.384344
180.000000	0.386982
181.000000	0.389611
182.000000	0.392232
183.000000	0.394846
184.000000	0.397451
185.000000	0.400047
186.000000	0.402635
187.000000	0.405215
188.000000	0.407786
189.000000	0.410349
190.000000	0.412903
191.000000	0.415448
192.000000	0.417985
193.000000	0.420512
194.000000	0.423031
195.000000	0.425541
196.000000	0.428042
197.000000	0.430533
198.000000	0.433016
199.000000	0.435489
200.000000	0.437954
201.000000	0.440409
202.000000	0.442854
203.000000	0.445291
204.000000	0.447718
205.000000	0.450136
206.000000	0.452544
207.000000	0.454943
208.000000	0.457332
209.000000	0.459712
210.000000	0.462083
211.000000	0.464443
212.000000	0.466795
213.000000	0.469137
214.000000	0.471469
215.000000	0.473791
216.000000	0.476104

OMEGA	EVAL
217.000000	0.478407
218.000000	0.480701
219.000000	0.482985
220.000000	0.485259
221.000000	0.487523
222.000000	0.489778
223.000000	0.492023
224.000000	0.494259
225.000000	0.496484
226.000000	0.498700
227.000000	0.500906
228.000000	0.503103
229.000000	0.505290
230.000000	0.507467
231.000000	0.509634
232.000000	0.511792
233.000000	0.513940
234.000000	0.516078
235.000000	0.518206
236.000000	0.520325
237.000000	0.522434
238.000000	0.524534
239.000000	0.526624
240.000000	0.528704
241.000000	0.530774
242.000000	0.532835
243.000000	0.534887
244.000000	0.536928
245.000000	0.538961
246.000000	0.540983
247.000000	0.542996
248.000000	0.545000
249.000000	0.546994
250.000000	0.548979
251.000000	0.550954
252.000000	0.552919
253.000000	0.554876
254.000000	0.556822
255.000000	0.558760
256.000000	0.560688
257.000000	0.562607
258.000000	0.564516
259.000000	0.566417
260.000000	0.568308
261.000000	0.570189
262.000000	0.572062
263.000000	0.573925
264.000000	0.575779
265.000000	0.577624
266.000000	0.579460
267.000000	0.581287
268.000000	0.583105
269.000000	0.584914

OMEGA	EVAL
270.000000	0.586714
271.000000	0.588505
272.000000	0.590287
273.000000	0.592060
274.000000	0.593824
275.000000	0.595580
276.000000	0.597326
277.000000	0.599064
278.000000	0.600793
279.000000	0.602514
280.000000	0.604226
281.000000	0.605929
282.000000	0.607623
283.000000	0.609309
284.000000	0.610987
285.000000	0.612656
286.000000	0.614316
287.000000	0.615968
288.000000	0.617612
289.000000	0.619247
290.000000	0.620874
291.000000	0.622493
292.000000	0.624103
293.000000	0.625705
294.000000	0.627299
295.000000	0.628885
296.000000	0.630463
297.000000	0.632032
298.000000	0.633594
299.000000	0.635147
300.000000	0.636693
301.000000	0.638231
302.000000	0.639760
303.000000	0.641282
304.000000	0.642796
305.000000	0.644302
306.000000	0.645801
307.000000	0.647291
308.000000	0.648774
309.000000	0.650250
310.000000	0.651718
311.000000	0.653178
312.000000	0.654630
313.000000	0.656075
314.000000	0.657513
315.000000	0.658943
316.000000	0.660366
317.000000	0.661781
318.000000	0.663189
319.000000	0.664590
320.000000	0.665984
321.000000	0.667370

OMEGA	EVAL
322.000000	0.668749
323.000000	0.670121
324.000000	0.671486
325.000000	0.672844
326.000000	0.674194
327.000000	0.675538
328.000000	0.676875
329.000000	0.678205
330.000000	0.679527
331.000000	0.680844
332.000000	0.682153
333.000000	0.683455
334.000000	0.684751
335.000000	0.686040
336.000000	0.687322
337.000000	0.688597
338.000000	0.689866
339.000000	0.691129
340.000000	0.692384
341.000000	0.693634
342.000000	0.694876
343.000000	0.696113
344.000000	0.697343
345.000000	0.698566
346.000000	0.699784
347.000000	0.700995
348.000000	0.702199
349.000000	0.703397
350.000000	0.704590
351.000000	0.705776
352.000000	0.706956
353.000000	0.708129
354.000000	0.709297
355.000000	0.710459
356.000000	0.711614
357.000000	0.712764
358.000000	0.713908
359.000000	0.715045
360.000000	0.716177
361.000000	0.717303
362.000000	0.718424
363.000000	0.719538
364.000000	0.720647
365.000000	0.721750
366.000000	0.722847
367.000000	0.723939
368.000000	0.725025
369.000000	0.726106
370.000000	0.727181
371.000000	0.728250
372.000000	0.729314
373.000000	0.730372

OMEGA	EVAL
374.000000	0.731425
375.000000	0.732473
376.000000	0.733515
377.000000	0.734552
378.000000	0.735583
379.000000	0.736610
380.000000	0.737630
381.000000	0.738646
382.000000	0.739657
383.000000	0.740662
384.000000	0.741662
385.000000	0.742658
386.000000	0.743648
387.000000	0.744633
388.000000	0.745613
389.000000	0.746588
390.000000	0.747558
391.000000	0.748523
392.000000	0.749483
393.000000	0.750438
394.000000	0.751389
395.000000	0.752335
396.000000	0.753275
397.000000	0.754211
398.000000	0.755143
399.000000	0.756069
400.000000	0.756991
401.000000	0.757909
402.000000	0.758821
403.000000	0.759729
404.000000	0.760633
405.000000	0.761532
406.000000	0.762426
407.000000	0.763316
408.000000	0.764201
409.000000	0.765082
410.000000	0.765959
411.000000	0.766831
412.000000	0.767699
413.000000	0.768562
414.000000	0.769421
415.000000	0.770276
416.000000	0.771126
417.000000	0.771973
418.000000	0.772815
419.000000	0.773652
420.000000	0.774486
421.000000	0.775315
422.000000	0.776141
423.000000	0.776962
424.000000	0.777779
425.000000	0.778592

OMEGA	EVAL
426.000000	0.779402
427.000000	0.780207
428.000000	0.781008
429.000000	0.781805
430.000000	0.782598
431.000000	0.783387
432.000000	0.784173
433.000000	0.784954
434.000000	0.785732
435.000000	0.786505
436.000000	0.787275
437.000000	0.788042
438.000000	0.788804
439.000000	0.789563
440.000000	0.790318
441.000000	0.791069
442.000000	0.791817
443.000000	0.792561
444.000000	0.793301
445.000000	0.794038
446.000000	0.794771
447.000000	0.795500
448.000000	0.796226
449.000000	0.796949
450.000000	0.797668
451.000000	0.798383
452.000000	0.799095
453.000000	0.799804
454.000000	0.800509
455.000000	0.801211
456.000000	0.801909
457.000000	0.802604
458.000000	0.803295
459.000000	0.803984
460.000000	0.804668
461.000000	0.805350
462.000000	0.806029
463.000000	0.806704
464.000000	0.807375
465.000000	0.808044
466.000000	0.808710
467.000000	0.809372
468.000000	0.810031
469.000000	0.810687
470.000000	0.811340
471.000000	0.811989
472.000000	0.812636
473.000000	0.813280
474.000000	0.813920
475.000000	0.814557
476.000000	0.815192
477.000000	0.815823

OMEGA	EVAL
478.000000	0.816452
479.000000	0.817077
480.000000	0.817700
481.000000	0.818319
482.000000	0.818936
483.000000	0.819550
484.000000	0.820160
485.000000	0.820768
486.000000	0.821374
487.000000	0.821976
488.000000	0.822575
489.000000	0.823172
490.000000	0.823766
491.000000	0.824357
492.000000	0.824945
493.000000	0.825531
494.000000	0.826114
495.000000	0.826694
496.000000	0.827271
497.000000	0.827846
498.000000	0.828418
499.000000	0.828988
500.000000	0.829554
501.000000	0.830119

STOP END OF PROGRAM AT STATEMENT 00 6 + 00 LINES

OMEGA	EVAL
500.000000	0.829554
600.000000	0.875128
700.000000	0.905112
800.000000	0.925698
900.000000	0.940361
1000.000000	0.951139
1100.000000	0.959273
1200.000000	0.965554
1300.000000	0.970499
1400.000000	0.974459
1500.000000	0.977678
1600.000000	0.980328
1700.000000	0.982535
1800.000000	0.984392
1900.000000	0.985969
2000.000000	0.987320
2100.000000	0.988485
2200.000000	0.989497
2300.000000	0.990382
2400.000000	0.991160
2500.000000	0.991847
2600.000000	0.992458
2700.000000	0.993002
2800.000000	0.993490
2900.000000	0.993929
3000.000000	0.994324
3100.000000	0.994683
3200.000000	0.995008
3300.000000	0.995305
3400.000000	0.995576
3500.000000	0.995824
3600.000000	0.996052
3700.000000	0.996261
3800.000000	0.996455
3900.000000	0.996634
4000.000000	0.996799
4100.000000	0.996953
4200.000000	0.997096
4300.000000	0.997229
4400.000000	0.997354
4500.000000	0.997470
4600.000000	0.997578
4700.000000	0.997680
4800.000000	0.997775
4900.000000	0.997865
5000.000000	0.997949
5100.000000	0.998029

STOP END OF PROGRAM AT STATEMENT 00 6 + 00 LINES

OMEGA	EVAL
5000.000000	0.997949
6000.000000	0.998575
7000.000000	0.998953
8000.000000	0.999198
9000.000000	0.999366
10000.000000	0.999486
11000.000000	0.999576
12000.000000	0.999643
13000.000000	0.999696
14000.000000	0.999738
15000.000000	0.999772
16000.000000	0.999799
17000.000000	0.999822
18000.000000	0.999841
19000.000000	0.999858
20000.000000	0.999872
21000.000000	0.999884
22000.000000	0.999894
23000.000000	0.999903
24000.000000	0.999911
25000.000000	0.999918
26000.000000	0.999924
27000.000000	0.999930
28000.000000	0.999934
29000.000000	0.999939
30000.000000	0.999943
31000.000000	0.999947
32000.000000	0.999950
33000.000000	0.999953
34000.000000	0.999956
35000.000000	0.999958
36000.000000	0.999960
37000.000000	0.999962
38000.000000	0.999964
39000.000000	0.999966
40000.000000	0.999968
41000.000000	0.999969
42000.000000	0.999971
43000.000000	0.999972
44000.000000	0.999973
45000.000000	0.999975
46000.000000	0.999976
47000.000000	0.999977
48000.000000	0.999978
49000.000000	0.999979
50000.000000	0.999979
51000.000000	0.999980
52000.000000	0.999981
53000.000000	0.999982
54000.000000	0.999982
55000.000000	0.999983
56000.000000	0.999984

RC NOTCH FILTERS OF THE GOLDMAN TYPE

by

CHIN-PANG YU

B. S., Taiwan Provincial Taipei Institute
of Technology, 1958

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Electrical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1967

RC notch filters with equal amplitudes and unequal amplitudes at zero end infinite frequencies in the frequency response curve are described in the literature. Their mein applications are in feedback amplifier problems.

Three notch filters were investigated in this paper: the Goldman type RC notch filter, a more general notch filter of the Goldmen type, and a still more complex RC notch filter. The parameter values for the exiatence of a notch frequency were found for each network by requiring that the capacitance heve e maximum value in eech filter. In the third case the capacitance wes e function of two varieble s, resulting in an interesting optimization problem.

The first and third networks were found to be equal amplitudes filters, wherees the second wes found to be en unequel amplitudes filter. The quality factor, Q, for the third network was found to be lower than that of the Goldman type filter; thus the more complex network features a wider bandwidth than the simple Goldman type notch network.